$\frac{2}{2}$

①

$\boxed{\text{Disclaimer: There may be errors!}}$

11/11/00 Solutions to practice problems (set 1)

(1) (a) $\frac{1+j1}{1-j1}$

$= \frac{(1+j)}{(1-j)} \times \frac{(1+j)}{(1+j)}$

$= \frac{\cancel{1}+\cancel{j^2}+2j}{1^2 - j^2} = \frac{2j}{2} = 1j = 0+j$ in cartesian

$= 1e^{jn\pi/2}$ in polar

$n = 1, 5, 9, \ldots$

(b) $(1+j)(1-j) = 1^2 - j^2$

$= 1^2 + 1 = 2 = 2+0j$ in cartesian

$2e^{j(2n\pi)}$ in polar,

$n \in$ Integers.

(c) $\frac{-1-j}{2+3j} = \frac{(-1-j)(2-3j)}{(2+3j)(2-3j)}$

$= \frac{-2+3j-2j+3j^2}{2^2-(3j)^2} = \frac{-5+j}{13}$

$\Rightarrow -5 + j$ (in cartesian)

$\Rightarrow$ (or $\sqrt{-5^2 + 1^2}\, e^{j \tan^{-1}(1/-5)}$

$\approx \underline{\underline{5.1\, e^{-j111.3^\circ}}}$ (in polar). [one solution is enough]

(2) $A\cos(\omega t + \phi) = \cos(2\pi 1000 t + \pi/4)$
$+ 2\sin(2\pi 1000 t + \pi/4)$

Now $\omega = 2\pi 1000 \frac{\text{rad}}{\text{sec}}$

$$A = \sqrt{\left(\sum_k A_k \cos\phi_k\right)^2 + \left(\sum_k A_k \sin\phi_k\right)^2}$$

$$= \sqrt{\left(1\cdot\cos\left(\frac{\pi}{4}\right) + 2\cos\left(\pi/4\right)\right)^2 + \left(1\sin\pi/4 + 2\sin\pi/4\right)^2}$$

$$\boxed{A \approx 1.73}$$

$$\phi = \tan^{-1}\frac{\sum_k A_k \sin\phi_k}{\sum_k A_k \cos\phi_k} = \tan^{-1}\frac{\cancel{3}\sin\pi/4}{\cancel{3}\cos\pi/4}$$

$$= \tan^{-1}(\tan\pi/4) \Rightarrow \boxed{\phi = \pi/4}$$

(3) $x(t) = \cos\sqrt{3}t + \sin 2\sqrt{3}t + \cos\left(3\sqrt{3}t + \frac{5\pi}{6}\right)$

Periodic, since ratio of the frequencies is a rational no.

Let $\omega_1 = \sqrt{3}$ rad/sec

$\omega_2 = 2\sqrt{3}$ rad/sec

$\omega_3 = 3\sqrt{3}$ rad/sec

$\frac{\omega_1}{\omega_2} = 1/2$, $\frac{\omega_2}{\omega_3} = \frac{2}{3}$, $\frac{\omega_1}{\omega_3} = \frac{1}{3}$, all are rational nos.

$\therefore \omega_0 = \gcd(\omega_1, \omega_2, \omega_3) = \sqrt{3}$ rad/sec

$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{3}}$ sec

check ur answer in your calculator,

If $x(t) = x(t+T)$.

check ~~the~~ if: $x(1) = x\left(1 + \frac{2\pi}{\sqrt{3}}\right)$

$$\left[x(1) = x\left(1 + \frac{2\pi}{\sqrt{3}}\right) \simeq -0.437\right]$$

(b) $y(n) = \cos(3\pi n) + \sin(12\pi n)$

Again, $\omega_1 = 3\pi$ rad/sample

$\omega_2 = 12\pi$ rad/sample

in order to find the period, lets use the definition of a periodic function:

$y(n+pk) = y(n)$

$\Rightarrow$ ~~$y(n+$~~ $\cos(3\pi(n+pk)) + \sin(12\pi(n+kp)) =$

$\cos(3\pi n) + \sin(12\pi n)$

$\Rightarrow \cos(3\pi n + 3\pi pk) + \sin(12\pi n + 12\pi kp)$

$= \cos(3\pi n) + \sin(12\pi n)$

$\Rightarrow \quad 3\pi pk = m 2\pi \quad -(1)$

$12\pi pk = n 2\pi \quad -(2)$

$\frac{(1)}{(2)} \Rightarrow \frac{m}{n} = \frac{1}{4} \Rightarrow \underline{\underline{n = 4m}}$

$\therefore \quad p = \frac{n 2\cancel{\pi}}{12\cancel{\pi}}$ | Let $k=1$, fundamental period |

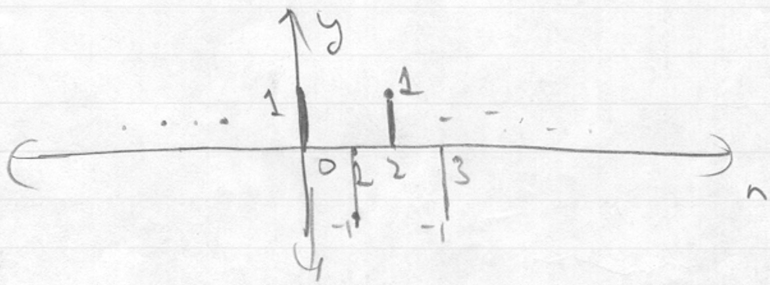
$p = n/6$

$\Rightarrow$ Smallest $p = 2$ samples $\left[\begin{array}{l} n = 12, \\ m = 3, \end{array} \right.$ $\left| \begin{array}{l} \frac{m}{n} \in \text{Rational}, \\ m \text{ and } n \text{ have} \\ \text{to be integers} \end{array} \right]$

$\therefore \omega_0 = \pi$ rad/sample

(Again, check ur answer using ur calculator)

This question is much more easier if you do it graphically:



Another method $\Rightarrow y(n) = \cos(3\pi n) + \cancel{\sin(12\pi n)}$ always 0 for $n \in$ Integers

$$y(n) = (-1)^n$$

$$\boxed{\text{period} = 2 \text{ samples}}$$

(4) (a) Now, $\cos(\omega_0 n + \theta) = \cos[(\omega_0 + 2\pi)n + \theta]$

$\downarrow$

If $|\omega_0| \le \pi$ $\quad \omega,$

(b)

(c)

(b) $\cos(\omega t + \theta) = \cos(\omega_0 t + \theta)$

$\Rightarrow \cos(\omega t)\cos\theta - \sin$

(b) <u>Given:</u>

$$X_k = \frac{1}{p}\int_0^p x(t)e^{-ik\omega_0 t}\,dt$$

(a) $y(t) = x(t-\tau)$

Now, $y(t+T) = y(t)$ if y is periodic with a period T.

$$\therefore\ y(t+T) = x(t+T-\tau)$$

$$= x(t-\tau+T)$$

$$= x(t-\tau) \quad [\text{If } T=p,\ x(t) \text{ is periodic with period } p]$$

$$\Rightarrow \boxed{T = p\ \text{secs}}$$

$$Y_k = \frac{1}{p}\int_0^p y(t)e^{-ik\omega_0 t}\,dt$$

$$= \frac{1}{p}\int_0^p x(t-\tau)e^{-ik\omega_0 t}\,dt$$

Let $t-\tau = u \Rightarrow t = u+\tau$

$\Rightarrow dt = du$

$$Y_k = \frac{1}{p}\int_{u=-\tau}^{-\tau+p} x(u)\, e^{-ik\omega_0(u+\tau)}\, du$$

$$= \underbrace{\left[\frac{1}{p}\int_{\substack{\text{over}\\ \text{one } p}} x(u)\, e^{-ik\omega_0 u}\, du\right]}_{\Downarrow \atop X_k} e^{-ik\omega_0\tau}$$

$$\Rightarrow \boxed{Y_k = X_k\, e^{-ik\omega_0\tau}}$$

(b) Now, $y(t+T) = y(t)$, T is a period.

$$\text{LHS} = y(t+T)$$

$$= x[a(t+T)] \qquad [y(t) = x(at)]$$

$$= x(at + aT) \text{ ——— } (1)$$

Now, $x(t) = x(t+p)$ [given]

$$\Rightarrow x(at) = x(at+p) \text{ — } (2)$$

$\therefore$ Comparing (1) & (2),

If $p = aT \Rightarrow \boxed{T = p/a}$