**EECS 20. Final Exam**
**December 20, 2001.**

Please use these sheets for your answer. Use the backs if necessary. **Write clearly and show your work.** Please check that you have 13 numbered pages.

Print your name and lab time below

Name: ______________________________

Lab time: ____________________________

Problem 1 (25):

Problem 2 (15):

Problem 3 (15):

Problem 4 (40):

Problem 5 (20):

Total:

1. **25 points** Please indicate whether the following statements are true or false. There will be no partial credit. They are either true or false. So please be sure of your answer.

   (a) $[\{1,2,3\} \to \{1,2,3\}] \subset [\{1,2,3\} \to \textit{Naturals}]$

   (b) Given a function $g\colon X \to Y$, $\textit{graph}(g) \subset X \times Y$.

   (c) Consider two identity functions:

   $$I_1\colon \{1,2,3\} \to \{1,2,3\}$$

   $$I_2\colon \textit{Naturals} \to \textit{Naturals}$$

   such that $\forall\, k \in \{1,2\}$ and $\forall\, n \in \textit{domain}(I_k)$, $I_k(n) = n$. Then $\textit{graph}(I_1) \subset \textit{graph}(I_2)$.

(d) $\{1,2,3\} \subset P(\{1,2,3\})$, where $P(X)$ is the powerset of $X$.

(e) $X \in P(X \times Y)$, where $X$ and $Y$ are sets and $P$ again denotes the powerset.

2. **15 points.** Consider a state machine $S$ where

$$\begin{aligned} \textit{Inputs} &= \{0, 1, \textit{absent}\} \\ \textit{Outputs} &= \{0, 1, \textit{absent}\} \\ \textit{States} &= \{a, b\} \\ \textit{initialState} &= a \end{aligned}$$

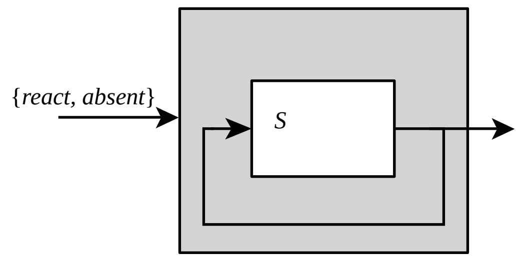
and the *update* function is such that

$$\begin{aligned} \textit{update}(a, 1) &= (a, 0) \\ \textit{update}(a, 0) &= (b, 1) \\ \textit{update}(b, 1) &= (b, 0) \\ \textit{update}(b, 0) &= (a, 1). \end{aligned}$$

(a) Draw the state transition diagram.

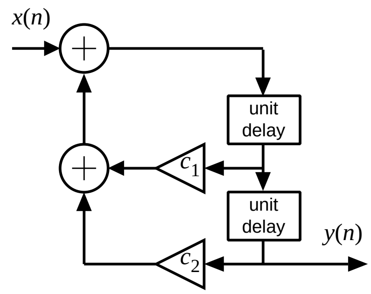
(b) Draw the state transition diagram for a simpler state machine that is bisimilar.

(c) Consider the feedback composition below:




Is this well-formed? Justify your answer.

3. **15 points.** Consider the system below:




The triangular symbols represent systems where the output is simply the input scaled by a constant $c_1$ or $c_2$.

(a) Write a difference equation relating $x$ and $y$. That is, give the relationship in the form

$$y(n) + a_1 y(n-1) + \cdots + a_M y(n-M) = b_0 x(n) + b_1 x(n-1) + \cdots + b_N x(n-N).$$

(b) Give $A, b, c, d$ for a state-space model for this system. **Hint**: The outputs of the unit delays are reasonable choices for state variables.

(c) Find the frequency response.

4. **40 points** Consider the continuous-time signal

$$\forall\, t \in \textit{Reals}, \quad x(t) = \sin(\pi t) + \cos(1.5\pi t),$$

where the units of $t \in \textit{Reals}$ is seconds.

(a) Find the fundamental frequency. Give the units.

(b) Find the Fourier series coefficients $A_0, A_1, \cdots$ and $\phi_1, \phi_2, \cdots$ in

$$\forall\, t \in \textit{Reals}, \quad x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k).$$

(c) Find the Fourier series coefficients $X_k$, $k \in \textit{Integers}$, in

$$\forall\, t \in \textit{Reals}, \quad x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t}.$$

(d) Let $y$ be the result of sampling this signal with sampling frequency $f_s = 1$ sample/second. Find the fundamental frequency for $y$, and give the units.

(e) For the same $y$, find the discrete-time Fourier series coefficients, $A_0, A_1, \cdots, A_K$ and $\phi_1, \cdots, \phi_K$ in

$$\forall\, n \in \textit{Naturals}, \quad y(n) = A_0 + \sum_{k=1}^{K} A_k \cos(k\omega_0 n + \phi_k)$$

where

$$K = \begin{cases} (p-1)/2 & \text{if } p \text{ is odd} \\ p/2 & \text{if } p \text{ is even} \end{cases}$$

where $p$ is the period. Be sure to note the limit $K$, and give the coefficients indexed from 0 to $K$.

(f) For the same $y$, find the discrete-time Fourier series coefficients, $Y_0, Y_1, \cdots, Y_{p-1}$ in

$$y(n) = \sum_{k=0}^{p-1} Y_k e^{ik\omega_0 n},$$

where $p$ is the period. Be sure to note the limits of the summation and to give the coefficients indexed from 0 to $p-1$.

(g) Find

$$w = \mathit{IdealInterpolator}_T(\mathit{Sampler}_T(x))$$

for $T = 1$ second.

(h) Give a lower bound on the sampling frequency that avoids aliasing distortion.

5. **20 points** Consider a continuous-time LTI system with input $x$ and output $y$ related by

$$\forall\, t \in \textit{Reals}, \quad y(t) = \int_{t-1}^{t+1} x(s) ds.$$

(a) Find the frequency response. Simplify! The following integration formula may be useful,

$$\int_a^b e^{c\omega} c\, d\omega = e^{cb} - e^{ca},$$

where $c \in \textit{Complex}$ is a constant.

(b) Find all real-valued sinusoidal input signals $x$ that yield output $y$ such that

$$\forall\, t \in \textit{Reals}, \quad y(t) = 0.$$

(c) Find and sketch the impulse response.

(d) Is the system causal?

**Use this page for overflow**