EECS20n, Quiz 5, 12/3/01

The quiz will take 15 minutes. Write your reponse on the sheet. Use the back if you need more space.

Please print your name here:

Last Name _____ First _____ Lab _____

Consider a continuous-time system with input signal x and output signal y where

$$\forall\, t \in Reals, \quad y(t) = x(t)e^{i\omega_1 t}$$

where $\omega_1 > 0$ is a real number.

1. Give the Fourier transform Y of y in terms of the Fourier transform X of x.

   Solution:

$$
\begin{aligned}
Y(\omega) &= \int_{-\infty}^{\infty} y(t)e^{-i\omega t}dt \\
&= \int_{-\infty}^{\infty} x(t)e^{i\omega_1 t}e^{-i\omega t}dt \\
&= \int_{-\infty}^{\infty} x(t)e^{-i(\omega-\omega_1)t}dt \\
&= X(\omega - \omega_1).
\end{aligned}
$$

2. Is the system linear?

   Solution: Yes.

3. Is the system time invariant?

   Solution: No.

For reference, the continuous-time Fourier transform relation is

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt.$$

The inverse relation is

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega.$$