EECS20n, Quiz 2

Solution

1. Let $f: Reals^2 \rightarrow Reals^2$ be a function where $\forall\ (x_1, x_2) \in Reals^2$,

$$f(x_1, x_2) = (y_1, y_2)$$

where

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Please indicate whether the following statements are true or false. There will be no partial credit, so please consider your answer carefully.

   (a) f is onto.   **false**

   (b) f is one-to-one   **false**

   (c) f is linear   **true**

2. Consider a SISO system with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad d = 0.$$

Find the zero-state impulse response.

Solution:

$$s(n+1) = \begin{bmatrix} s_1(n+1) \\ s_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(n)$$

From this, we can determine that if the input is given by $\forall\ n \in Integers, \quad x(n) = \delta(n)$, then

$$s(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad s(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad s(2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad s(3) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and

$$s(n) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for all $n \geq 3$. Using the output formula,

$$y(n) = [0\ 1] \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix} + 0,$$

we can determine that

$$y(0) = 0, \quad y(1) = 1, \quad y(2) = 0, ...$$

and so

$$y(n) = \delta(n-1).$$