EECS 20. Final Exam. December 21, 2004. Use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab day and time below

Name: ____

Lab day and time: ______________________________________

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Problem 6:

Total:

1. **15 points, 3 points each part** Give precise definitions of the following:

   (a) The space *Images* of all 600×900 images with pixel values in $\{0, 1, \cdots, 255\}$.

   (b) A one-to-one and onto function $f : [0, \infty) \to [0, 1)$.

   (c) A linear one-to-one function $f : \textit{Complex} \to \textit{Reals}^2$.

   (d) The spaces *ContSignals* and *DiscSignals* of continuous-time and discrete-time complex-valued signals (use the [ ] notation):

   Now define the system

   $$\textit{Sampler}_T : \textit{ContSignals} \to \textit{DiscSignals}$$

   which samples its input every T sec.

   (e) The convolution $z = x * y$ when

      i. $x, y : \textit{Reals} \to \textit{Reals}$.

      ii. $x, y : \textit{Ints} \to \textit{Reals}$.

2. **15 points** Design two state machines, both with *Inputs* $= \{0, 1\}$, *Outputs* $= \{0, 1, 2\}$, and with input-output functions S_1, S_2 given below.

(a) **7 points**

$$\forall x, \forall n \quad S_1(x)(n) = (n_1 - n_0) \bmod 3,$$

in which n_1 and n_0 are the numbers of 1's and 0's in $(x(0), \cdots, x(n))$, respectively.

(b) **8 points**

$$\forall x, \forall n \quad S_2(x)(n) = \begin{cases} 0, & \text{if } (n_1 - n_0) \text{ is even} \\ 1, & \text{if } (n_1 - n_0) \text{ is odd} \end{cases}$$

in which n_1, n_0 are as above.

3. **15 points, 5 points each part** Consider a LTI system with $[A, b, c, d]$ representation given by:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad c^T = [1 \ \ 0], \quad d = 0.$$

(a) Calculate the zero-input state response when the initial state is $s(0) = [s_1(0) \quad s_2(0)]^T$.

(b) Calculate the (zero-state) impulse response, h.

(c) Calculate the response $y(n), n \geq 0$ when the initial state is $s(0) = [1 \quad 1]^T$ and the input signal is $\forall n \geq 0, x(n) = \delta(n-1)$.

4. **20 points** Consider the difference equation

$$y(n) - 2y(n-1) = x(n) - 3x(n-1).$$

(a) **4 points** Take the state at time n as $s(n) = [y(n-1), x(n-1)]^T$ and write down the $[A, b, c, d]$ representation of the system.

(b) **4 points** Implement the difference equation using two delay elements whose outputs are the two state components.

(c) **6 points** Find another implementation using only *one* delay element and find the $[A, b, c, d]$ representation for this implementation.

(d) **6 points** Determine the zero-state impulse response.

5. **15 points** The bandwidth of a continuous time signal x with FT X is by definition the smallest frequency ω_B such that $X(\omega) = 0$ for $|\omega| > \omega_B$.

   (a) **3 points** What is the bandwidth of the signals: $\forall t \in$ *Reals*,

   $$x_k(t) = \sin(10k\pi t), k = 1, 2, 3; \quad x_4(t) = x_1(t) + x_2(t) + x_3(t).$$

   Specify the units of the bandwidth.

   (b) **3 points** What is the FT of $x_k, k = 1, \cdots, 4$?

   (c) **4 points** Suppose x_k is sampled at frequency $\omega_s = 30\pi$ rad/sec. Find a simple expression for the sampled signal y_k.

   (d) **5 points** Find signals z_k : *Reals* $\rightarrow$ *Reals* such that (i) the bandwidth of z_k is smaller than 15π rad/sec, which is one-half the sampling frequency; and (ii) if z_k is sampled at frequency ω_s it also yields the signal y_k.

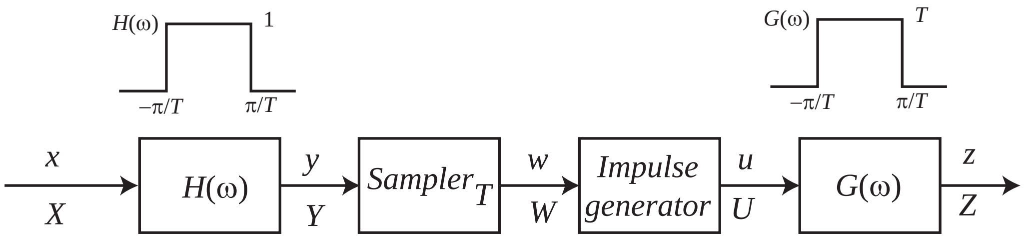

Figure 1: Setup for problem 6

6. **20 points, 5 points each part** Consider the setup of figure 1. The filters H, G are as shown; the sampling period is T seconds.

   (a) Express w, u in terms of y and W, U in terms of Y.

   (b) Express Z in terms of X.

   (c) Determine y and z for $T = 0.1s$ and $\forall t, x(t) = \sin(25\pi t) + \sin(5\pi t)$.

   (d) Suppose in this setup H is changed to $\forall \omega, H(\omega) = 1$. Take T, x as above, and determine z.

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