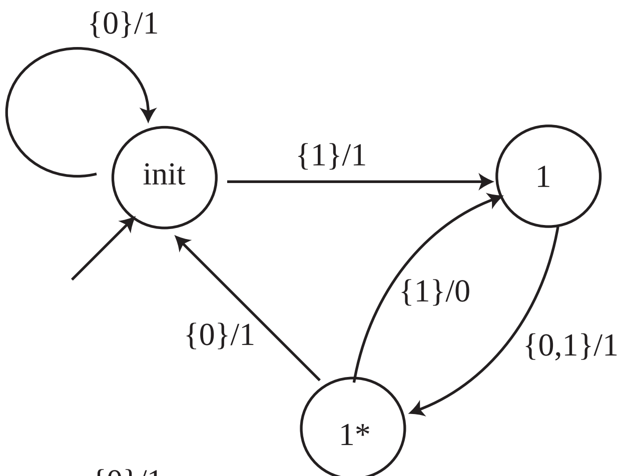
EECS 20. Midterm No. 1, Solution, October 7, 2004.

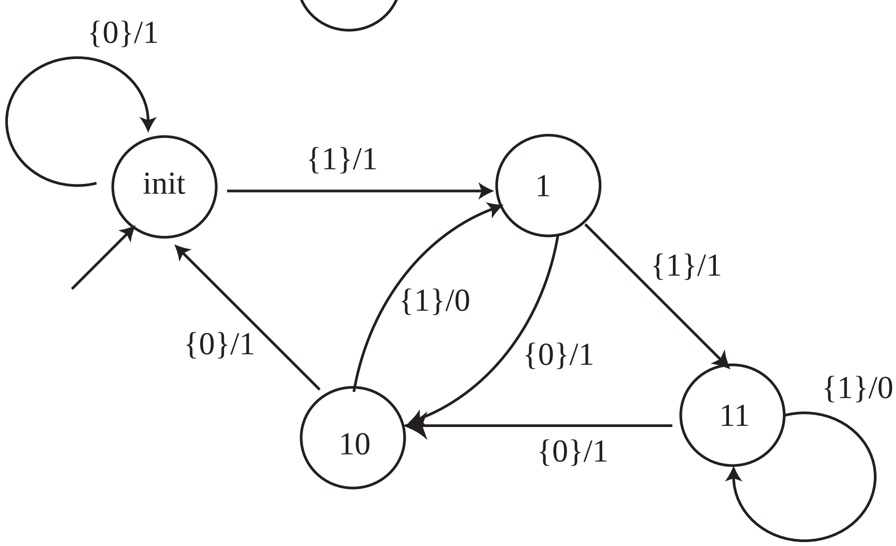
1. **10 points** Let $[Nats_0 \to \{0,1\}]$ be the input and output signal spaces. Construct a machine whose input-output function is:

$$\forall x, \forall n, F(x)(n) = \begin{cases} 0, & \text{if } (x(n-2), x(n)) = (1,1), \\ 1, & \text{otherwise} \end{cases}$$

Note $x(n-2)$ **in specification of** F**.**

The first machine below fails on input 1101. The second machine is correct.





2. **10 points. 2points for each part.**

(a) Using the notation $[X \to Y]$ for a signal space write down the signal space for:

i. *Voices*, comprising all analog voice signals of duration 1 second.

$$\mathit{Voices} = [[0,1] \to \mathit{Reals}]$$

ii. *SampledVoices*, comprising the signals obtained by sampling analog voice 8,000 times per second.

$$\mathit{SampledVoices} = [\mathit{DigiTime} \to \mathit{Reals}]$$

in which

$$\mathit{DigiTime} = \{0, 1/8000, 2/8000, \cdots, 7999/8000, 1\}$$

iii. *DigitalVoices*, comprising sampled voice signals whose magnitude is represented by an 8-bit integer.

$$\mathit{DigitalVoices} = [\mathit{DigiTime} \to \mathit{Integers}_8]$$

in which

$$\mathit{Integers}_8 = \{0, 1, \cdots, 255\}$$

iv. *Texts*, comprising the set of all English sentences.

$$\mathit{Texts} = [\mathit{Nats}_0 \to \mathit{EnglishWords}]$$

in which *EnglishWords* is the set of all English words.

(b) A *VoiceRecognizer* is a system that converts digial voice into text. What is the range and domain of this system?

The domain of *VoiceRecognizer* is *DigitalVoices* and its range is *Texts*.

3. **30 points. 5 points for each part.** Indicate whether the following statements are true or false. There is no partial credit.

(a) Suppose P, Q, R are true assertions. Then

$$\neg[[\neg P \vee Q] \wedge [P \vee [R \wedge \neg P]]] \text{ is true}$$

$\boxed{\text{False}}$

(b) If set A has 4 elements, its power set $P(A)$ has $4! = 24$ elements.

$\boxed{\text{False}}$, because $P(A)$ has $2^4 = 16$ elements.

(c) The function $f : [0, 1] \to [0, 1]$ given by

$$\forall x, \quad f(x) = e^{-x}$$

has a unique fixed point.

$\boxed{\text{True}}$, because the equation $x = e^{-x}$ has a unique solution.

(d) If sets X and Y have m and n elements respectively, the set $[X \to Y]$ has $m \times n$ elements.

$\boxed{\text{False}}$, because $[X \to Y]$ has n^m elements.

(e) There is no deterministic state machine with $Inputs = Outputs = \{0, 1\}$ whose input-output function F is given by: for all input signals x, the output signal $F(x)$ is

$$\forall n \in Nats_0, \quad F(x)(n) = x(n + 1) \tag{1}$$

$\boxed{\text{True}}$, because the input-output function of a deterministic machine must be causal, whereas F is not causal.

(f) There is a non-deterministic state machine with $Inputs = Outputs = \{0, 1\}$ whose *Behaviors* include $(x, F(x))$ for any input signal x, and $F(x)$ given by (1).

$\boxed{\text{True}}$, because there is a trivial one-state non-deterministic machine whose behaviors is *InputSignals* $\times$ *OutputSignals*.

4. **20 points. 10 points for each part.** Suppose A, B are non-deterministic state machines with *Inputs* and *Outputs* equal to $\{0, 1\}$,

$$A = (\mathit{States}_A, \mathit{possibleUpdates}_A, \mathit{init}_A)$$
$$B = (\mathit{States}_B, \mathit{possibleUpdates}_B, \mathit{init}_B)$$

Let C be the cascade composition of A and B.

C has the same *Inputs* and *Outputs* as A, B. Denote the other elements of C by

$$C = (\mathit{States}_C, \mathit{possibleUpdates}_C, \mathit{init}_C).$$

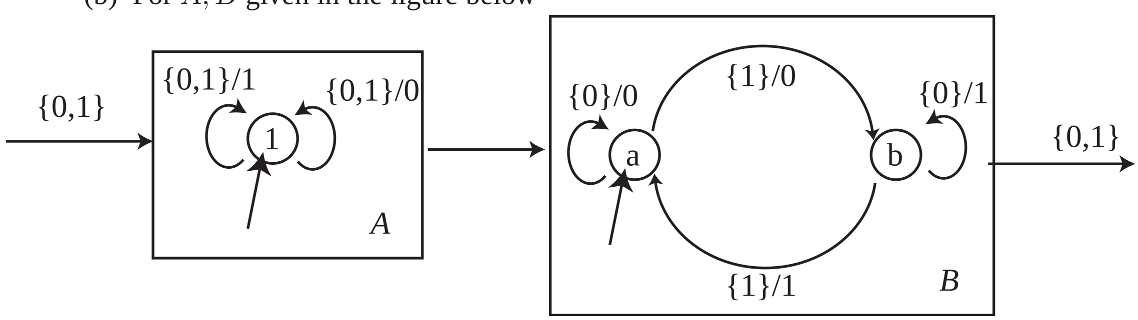
(a) Express these in terms of the elements of A, B.

$$\mathit{States}_C = \mathit{States}_A \times \mathit{States}_B$$
$$\mathit{init}_C = (\mathit{init}_A, \mathit{init}_B)$$

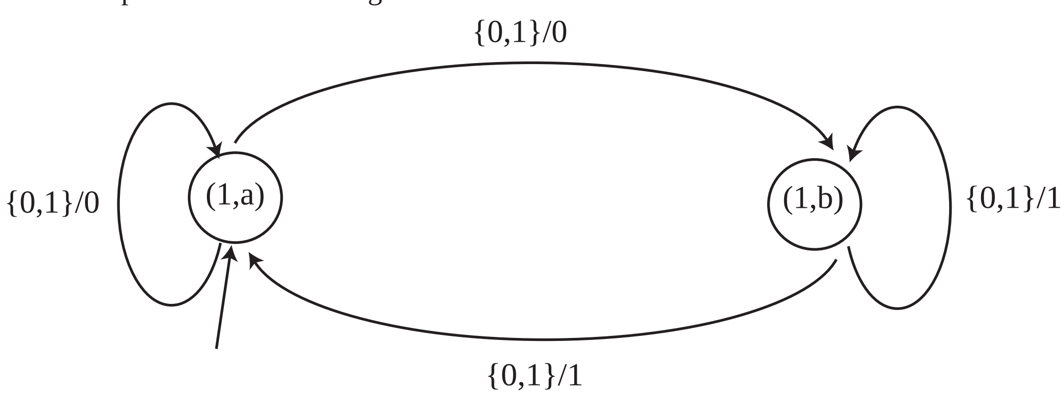
and $\forall (s_a, s_b) \in \mathit{States}_C, \forall x \in \mathit{Inputs}$,

$$\mathit{possibleUpdates}_C((s_a, s_b), x)) = \{(s'_a, s'_b), y') \mid$$
$$\exists y, (s'_a, y) \in \mathit{possibleUpdates}_A(s_a, x), (s'_b, y') \in \mathit{possibleUpdates}_B(s_b, y')\}$$

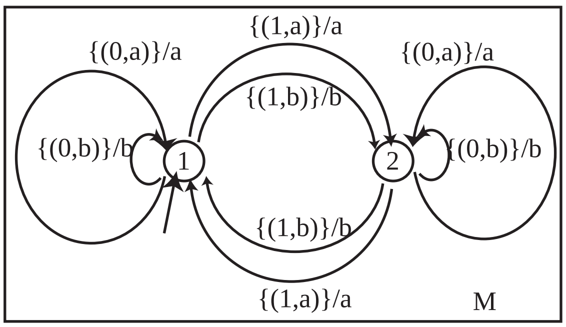
(b) For A, B given in the figure below

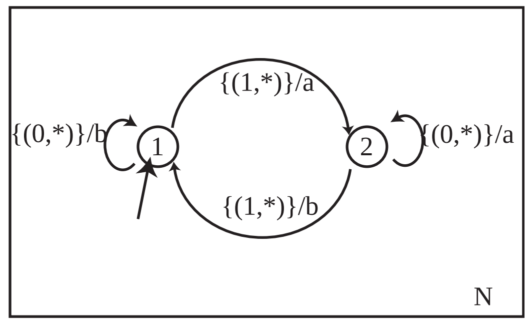


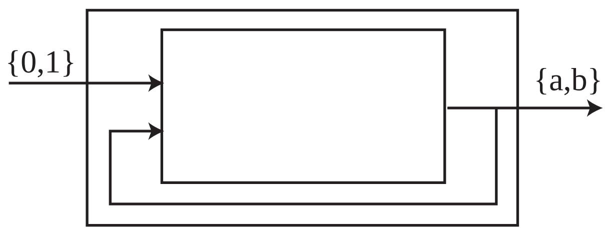
express C as a state diagram.



5. **20 points. 10 points each part.** M and N are machines with *Input* $= \{0, 1\} \times \{a, b\}$ and *Outputs* $= \{a, b\}$.







Feedback composition

(a) Suppose a feedback connection is placed around M as shown above. Is the resulting composition well-formed? If it is, draw the transition diagram for the composite machine below.

The feedback composition with M is not well-formed.

(b) Repeat part 5a for N.

The feedback composition with N is well-formed and given below: