EECS20n, Quiz 6 Solution, 11/2/04

1. Let $g : \textit{Ints} \to \textit{Reals}$ be any signal. Let $p \in \textit{Ints}$ and define $f : \textit{Ints} \to \textit{Reals}$ by

$$\forall n, \quad f(n) = \sum_{k=-\infty}^{\infty} g(n - kp). \tag{1}$$

i. **2 points** Prove that f is periodic with period p.

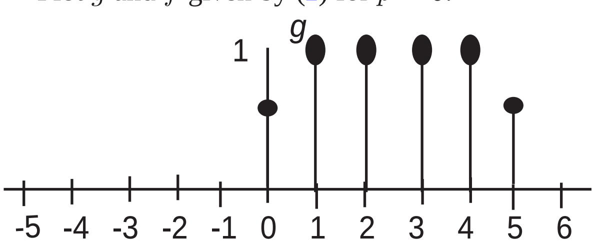
Proof $\forall n$,

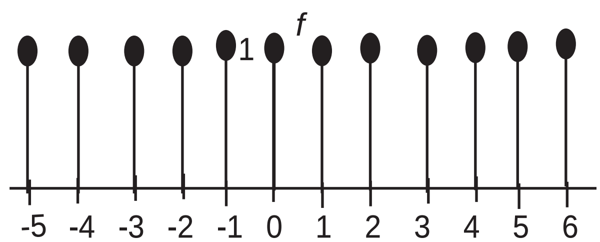
$$f(n+p) = \sum_{k=-\infty}^{\infty} g(n+p-kp) = \sum_{k=-\infty}^{\infty} g(n-(k-1)p) = f(n)$$

ii. **2 points** Suppose g is given by

$$\forall n, \quad g(n) = \begin{cases} 0.5, & n = 0, 5 \\ 1, & 1 \le n \le 4 \\ 0, & n > 5 \end{cases}$$

Plot g and f given by (1) for $p = 5$.





2. **3 points** For the signals $x : \textit{Ints} \to C$ given below, determine if x is periodic (Y or N); and if it is periodic, determine its period.

$\forall n, x(n) =$	x is periodic (Y or N)	period of x is
$e^{i\frac{2}{5}\pi n}$	Y	5
$e^{i\frac{2}{5}\pi n} + e^{i\frac{2}{3}\pi n}$	Y	15
$e^{i\sqrt{2}\pi n}$	N	

3. Suppose a differentiable periodic signal f has the Fourier Series representation

$$\forall t \in \textit{Reals}, \quad f(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k).$$

Its derivative g is also periodic with Fourier Series representation: $\forall t$

$$\forall t \in \textit{Reals}, \quad g(t) = \frac{df}{dt} = 0 + \sum_{k=1}^{\infty} -k\omega_0 A_k \sin(k\omega_0 t + \phi_k) = \sum_{k=1}^{\infty} k\omega_0 A_k \cos(k\omega_0 t + \frac{\pi}{2} + \phi_k)$$

using $-\sin\alpha = \cos(\frac{\pi}{2} + \alpha)$.

3 points Determine B_0, B_k, θ_k in terms of A_0, A_k, ϕ_k:

$$B_0 = 0, \quad B_k = k\omega_0 A_k, \quad \theta_k = \frac{\pi}{2} + \phi_k$$