

## EECS20n, Quiz 7 Solution, 11/16/04

1. **5 points** Find the frequency response  $H$  for the difference equation

$$y(n) = y(n-1) + x(n).$$

**Answer**

$$\forall \omega, \quad H(\omega) = \frac{1}{1 - e^{i\omega}}.$$

Find a difference equation whose frequency response is

$$\forall \omega, \quad H(\omega) = \frac{1 + 2e^{-i\omega} + e^{-i2\omega}}{1 + e^{-i\omega}}$$

**Answer**

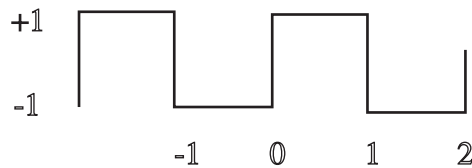
$$y(n) + y(n-1) = x(n) + 2x(n-1) + x(n-2).$$

2. **5 points** For the periodic signal  $x$  of the figure below determine its exponential Fourier Series,

$$\forall t, \quad x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t}.$$

**Hint** The coefficients are (period  $p, \omega_0 = 2\pi/p$ ):

$$X_k = \frac{1}{p} \int_0^p x(t) e^{-ik\omega_0 t} dt.$$



**Answer** The period is  $p = 2$  sec and  $\omega_0 = \pi$  rad/sec. So

$$\begin{aligned} X_k &= \frac{1}{2} \int_{-1}^1 x(t) e^{-ik\pi t} dt = \frac{1}{2} \left[ - \int_{-1}^0 e^{-ik\pi t} dt + \int_0^1 e^{-ik\pi t} dt \right] \\ &= \frac{1}{2} \left[ - \frac{1}{-ik\pi} e^{-ik\pi t} \Big|_{-1}^0 + \frac{1}{-ik\pi} e^{-ik\pi t} \Big|_0^1 \right] \\ &= \frac{1}{-ik\pi} [e^{ik\pi} - 1] \\ &= \begin{cases} 0, & k \text{ even} \\ \frac{2}{ik\pi}, & k \text{ odd} \end{cases} \end{aligned}$$