EECS20n, Quiz 8 Solution, 12/02/04

1. An audio signal x has Fourier Transform X such that $X(\omega) = 0, |\omega| > 2\pi \times 10,000$ rad/sec. The transmitted signal is $y(t) = \cos(2\pi \times f_c t) \times x(t)$, in which the carrier frequency is $f_c = 100,000$ Hz.

1. **5 points** For what values of ω is $Y(\omega) = 0$?
2. **5 points** An AM receiver constructs the signal $z(t) = y(t) \times \cos(2\pi f_c t)$. Express z in terms of x.

Answer 1. Let $\omega_0 = 20,000 \times \pi$. Then,

$$Y(\omega) = \frac{1}{2}[X(\omega - \omega_0) + X(\omega + \omega_0)],$$

so

$$Y(\omega) = 0, \quad \text{unless } |\omega - \omega_0| \leq 10,000\pi \text{ or } |\omega + \omega_0| \leq 10,000\pi.$$

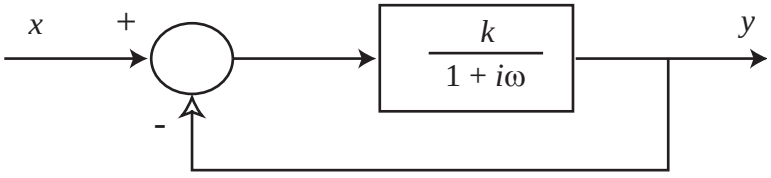
2. Directly from $z(t) = x(t)[\cos(\omega_0 t)]^2$ or from

$$\begin{aligned} Z(\omega) &= \frac{1}{2}[Y(\omega - \omega_0) + Y(\omega + \omega_0)] \\ &= \frac{1}{2}X(\omega) + \frac{1}{4}[X(\omega - 2\omega_0) + X(\omega + 2\omega_0], \end{aligned}$$

one gets

$$z(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t) \times \cos(2\omega_0 t).$$

2. **10 points** Consider the feedback system below. First find the frequency response H and then the impulse response h for $k = 1, 10, 100$. [**Hint** Recall $e^{-t}u(t) \leftrightarrow \frac{1}{1+i\omega}$ and the time change formula $x(at) \leftrightarrow \frac{1}{|a|}X(\frac{\omega}{a})$.



Answer The frequency response is

$$\forall \omega, \quad H(\omega) = \frac{k}{(k+1) + i\omega}.$$

and so the impulse response is

$$\forall t, \quad h(t) = ke^{-(k+1)t}u(t).$$