

EECS 20n, Diagnostic Takehome Exam, 8/31/04, Due next week in Lab

This 'takehome' exam is to provide feedback to you and to me about how well prepared you are with the prerequisite math for this course. You can consult your math textbook. The exam will take about one hour. **Write clearly, put a box around your answer, and show your work.**

Print: Last Name _____ First _____ Lab time _____ TA Name _____

1. For the function $f : \text{Reals} \rightarrow \text{Reals}, \forall x, f(x) = e^{t-5}$,

$$\frac{df}{dt}(t) = \quad , \int_0^t f(s)ds =$$

2. Let $z_1 = 3 + 4i$ and $z_2 = 5 + 12i$ be two complex numbers. Then

- (a) $z_1 + z_2 =$
- (b) $z_1 * z_2 =$
- (c) $z_2/z_1 =$

3. (a) $e^{i\pi} =$

- (b) Show why $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

- (c) Express $\sin 3\theta$ in terms of $\sin \theta$.

- (a) Does $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converge? Why?

- (b) What is $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} =$

4. Solve the following first order linear differential equation:

$$\frac{dy}{dx} = 2x + 1,$$

with the initial condition $y(0) = 0$. What is $y(1)$? Plot $y(x)$ for $0 \leq x \leq 1$.

5. Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

(a) Verify $A^2 = A$.

(b) Is it invertible? What is A^{-1} ?

(c) Find all its eigenvalues.