

LAST Name \_\_\_\_\_ FIRST Name \_\_\_\_\_

Lab Time \_\_\_\_\_

- (5 Points) Print your name and lab time in legible, block lettering in the appropriate spaces provided above.
- This quiz should take you up to 15 minutes to complete. You will be given at least 15 minutes—up to a maximum of 20 minutes—to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you will receive full credit for the problem *only if* you justify your answer and explain your work clearly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	
1	20	
2	20	
<b>Total</b>	<b>45</b>	

You may use this page for scratch work only.  
Without exception, subject matter on this page will *not* be graded.

**Problem 1 (20 Points)** Consider the set

$$X = [\{a\} \rightarrow \{1, 2, 3\}] .$$

(a) (5 Points) How many elements are in the set  $X$ ?  
Explicitly specify one element in  $X$ .

(b) (5 Points) How many elements are in the set  $Y = [X \rightarrow X]$ ?  
Explicitly specify one element in  $Y$ .

(c) (10 Points) By circling an appropriate answer choice, state whether each of the following assertions, considered independently of the other, is true or false. No explanation is necessary:

i.  $X \subset [\{a, b\} \rightarrow \{1, 2, 3\}]$ .

True

False

ii.  $X \subset [\{a\} \rightarrow \{1, 2, 3, 4\}]$ .

True

False

**Problem 2 (20 points)** Let  $X = [\mathbb{N}_0 \rightarrow \{0, 1\}]$ , where  $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$ .

Consider a system

$$F : X \rightarrow X$$
$$\forall x \in X, \forall n \in \mathbb{N}_0, (F(x))(n) = \begin{cases} 0 & \text{if } n \bmod 3 = 0 \\ 1 & \text{otherwise} \end{cases}.$$

(Note: The numbers 0, 3, 6, 9, ... are divisible by 3; therefore, they have mod 3 equal to zero. Modulo 3 numbers work as follows. For any  $n \in \mathbb{N}_0$ ,  $n \bmod 3 = k$ , where  $0 \leq k \leq 2$  is the unique number in  $\mathbb{N}_0$  such that 3 divides  $n - k$ . Thus, there are only three distinct modulo 3 numbers, namely, 0, 1, 2.)

(a) (6 Points) Is the system  $F$  memoryless? Justify your answer.

(b) (6 Points) Is the system  $F$  causal? Justify your answer.

- (c) (8 Points) Determine the input alphabet and the output alphabet for, and draw the state transition diagram of, a state machine that implements the system  $F$ . Ignore stuttering symbols, but clearly specify the initial state on the state transition diagram that you provide.

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