

LAST Name \_\_\_\_\_ FIRST Name \_\_\_\_\_

Lab Time \_\_\_\_\_

- (5 Points) Print your name and lab time in legible, block lettering in the appropriate spaces provided above.
- This quiz should take you up to 15 minutes to complete. You will be given at least 15 minutes—up to a maximum of 20 minutes—to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you will receive full credit for the problem *only if* you justify your answer and explain your work clearly.
- We hope you do a *fantastic* job on this quiz!

Problem	Points	Your Score
Name	5	
1	20	
2	20	
<b>Total</b>	<b>45</b>	

You may use this page for scratch work only.  
Without exception, subject matter on this page will *not* be graded.

**Q2.1 (20 Points)** For each system

$$F : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$$

described below, the input signal is denoted by  $x$  and the output signal by  $y$ . Determine which, if any, of the following properties each system possesses: **(I)** linearity, **(II)** time invariance, **(III)** causality, and **(IV)** memorylessness. Explain your reasoning clearly, but succinctly.

(a) [10 Points]  $y(n) = x(n^2)$ ,  $\forall n \in \mathbb{Z}$ .

(b) [10 Points]  $y(n) = \cos(x(n))$ ,  $\forall n \in \mathbb{Z}$ .

**Q2.2 (20 points)** Consider a causal, discrete-time, SISO LTI system having the following  $[A, B, C, D]$  state-space representation ( $\forall n \geq 0$ ):

$$\underbrace{\begin{bmatrix} s_1(n+1) \\ s_2(n+1) \end{bmatrix}}_{s(n+1)} = \underbrace{\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}}_A \underbrace{\begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix}}_{s(n)} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_B x(n)$$

$$y(n) = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_C s(n) + \underbrace{1}_D x(n).$$

The input signal, the output response, and the state response are  $x : \mathbb{N}_0 \rightarrow \mathbb{R}$ ,  $y : \mathbb{N}_0 \rightarrow \mathbb{R}$ , and  $s : \mathbb{N}_0 \rightarrow \mathbb{R}$ , respectively.

- (a) [10 Points] For this part only, suppose the input signal  $x$  is zero (i.e.,  $x(n) = 0, \forall n \geq 0$ ) and the initial state

$$s(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Determine a simple expression for  $s(n)$ , the state of the system at time  $n$  ( $\forall n \geq 1$ ).

(b) [10 Points] For (i) and (ii) below, assume that the initial state is zero, i.e.,

$$s(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} .$$

(i) [7 Points] Determine a pair of input signal values  $x(0)$  and  $x(1)$  so that the state of the system at time  $n = 2$  is given by:

$$s(2) = \begin{bmatrix} 3 \\ 0 \end{bmatrix} .$$

Is your answer unique? If so, explain. If not, specify another pair of input sample values  $x(0)$  and  $x(1)$  that produces the same target state  $s(2)$ .

(ii) [3 Points] Determine a pair of input signal values  $x(0)$  and  $x(1)$  so that

$$s(2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} ,$$

or explain (clearly, but succinctly) why no such  $x(0)$  and  $x(1)$  can be found.

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