EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 1
Department of Electrical Engineering and Computer Sciences 21 September 2006
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Vector FIRST Name Complex

Lab Time Anytime is Lab Time!

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2	10	10
3	10	10
Total	**45**	45

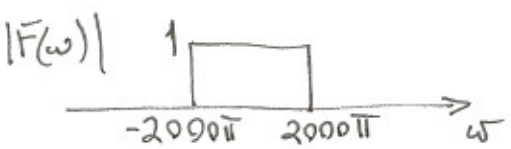
Q1.1 (20 Points) Consider a function $F : \mathbb{R} \to \mathbb{C}$ characterized as follows:

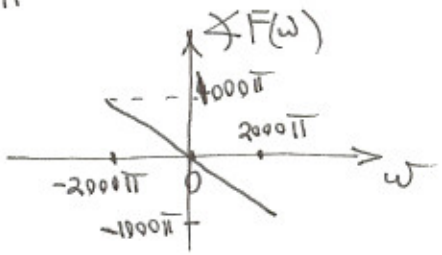
$$\forall \omega \in \mathbb{R}, \quad F(\omega) = \begin{cases} e^{-i\omega/2} & |\omega| \leq 2000\pi \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine an expression for—and sketch and label the salient features of the graphs of—the magnitude $|F(\omega)|$ and angle $\angle F(\omega)$ of function F, where $F(\omega) = |F(\omega)|\, e^{i\,\angle F(\omega)}$ is the complex polar representation of F.

$$|F(\omega)| = \begin{cases} |e^{-i\omega/2}| & |\omega| \leq 2000\pi \\ 0 & e/\omega \end{cases} = \begin{cases} 1 & |\omega| \leq 2000\pi \\ 0 & e/\omega \end{cases}$$

$$\angle F(\omega) = \begin{cases} -\omega/2 & |\omega| \leq 2000\pi \\ \text{undefined} & e/\omega \end{cases}$$





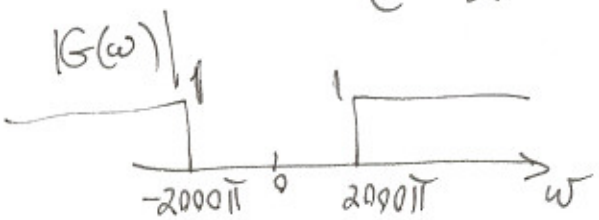
(b) A function $G : \mathbb{R} \to \mathbb{C}$ is related to the function F as follows:

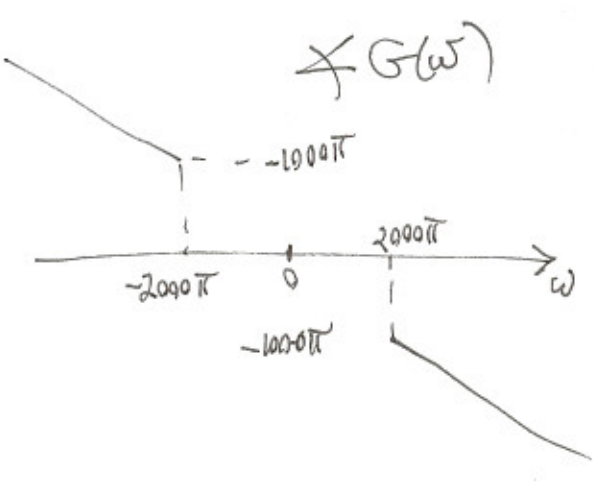
$$\forall \omega \in \mathbb{R}, \quad G(\omega) = e^{-i\omega/2} - F(\omega)\,.$$

Provide a clear sketch of the graphs of $|G(\omega)|$ and $\angle G(\omega)$, the magnitude and phase of G, respectively. Be sure to label the salient features of your graphs.

$$G(\omega) = \begin{cases} 0 & |\omega| \leq 2000\pi \\ e^{-i\omega/2} & e/\omega \end{cases} \Rightarrow |G(\omega)| = \begin{cases} 0 & |\omega| \leq 2000\pi \\ 1 & e/\omega \end{cases}$$

$$\angle G(\omega) = \begin{cases} \text{undefined} & |\omega| \leq 2000\pi \\ -\omega/2 & e/\omega \end{cases}$$





Q1.2 (10 Points) Consider the two-dimensional matrix function $H : \mathbb{R}^2 \to \mathbb{R}^2$ characterized as follows: $\forall x \in \mathbb{R}^2, y = H\,x$, where

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}.$$

The function H rotates the two-dimensional vector x by an angle θ. If $\theta > 0$, the rotation is counter-clockwise; if $\theta < 0$, the rotation is clockwise.

Without using any knowledge of linear algebra, and without any complicated mathematical manipulation, determine the inverse function H^{-1}; explain your thought process. Succinctly, but clearly and convincingly, explain whether H is one-to-one, onto, neither, or both.

H rotates by θ, so H^{-1} must rotate by $-\theta$. Hence,

$$H^{-1} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Since H is invertible (we constructed its inverse), then it must be onto and one-to-one.

Q1.3 (10 Points) Assume to be true that: "At least one EECS student is overworked"; *and* "Every overworked person is tired"; *and* "No Journalism student is overworked." For each independent assertion (a) and (b), select the strongest correct characterization from among the following choices. "The assertion must be true"; "The assertion could be true, but does not necessarily have to be true"; or "The assertion cannot be true." Explain your reasoning succinctly, but clearly and convincingly.

(a) At least one tired person must be an EECS student. Must be true.

At least one EECS student is overworked } $\Rightarrow$ At least one
Every overworked person is tired

EECS student is tired $\Rightarrow$ At least one tired person is an EECS student.

(b) No Journalism student is tired. Could be true, but not necessarily so

Let OW $\triangleq$ a person is overworked : OW $\Rightarrow$ T
T $\triangleq$ a person is Tired
$\overline{J}$ $\triangleq$ a person is a $\overline{\text{Journalism student}}$: J $\Rightarrow \neg$OW

From these two premises, OW $\Rightarrow$ T and J $\Rightarrow \neg$OW, we cannot conclude whether any Journalism student is tired or not. For example, a Journalism student may be tired for a reason other than being overworked, e.g., say from hunger.