

LAST Name _____ FIRST Name _____

Lab Time _____

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	
1	20	
2	10	
3	10	
Total	45	

Q1.1 (20 Points) Consider a function $F : \mathbb{R} \rightarrow \mathbb{C}$ characterized as follows:

$$\forall \omega \in \mathbb{R}, \quad F(\omega) = \begin{cases} e^{-i\omega/2} & |\omega| \leq 2000\pi \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Determine an expression for—and sketch and label the salient features of the graphs of—the magnitude $|F(\omega)|$ and angle $\angle F(\omega)$ of function F , where $F(\omega) = |F(\omega)| e^{i \angle F(\omega)}$ is the complex polar representation of F .

- (b) A function $G : \mathbb{R} \rightarrow \mathbb{C}$ is related to the function F as follows:

$$\forall \omega \in \mathbb{R}, \quad G(\omega) = e^{-i\omega/2} - F(\omega).$$

Provide a clear sketch of the graphs of $|G(\omega)|$ and $\angle G(\omega)$, the magnitude and phase of G , respectively. Be sure to label the salient features of your graphs.

Q1.2 (10 Points) Consider the two-dimensional matrix function $\mathbf{H} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ characterized as follows: $\forall \mathbf{x} \in \mathbb{R}^2, \mathbf{y} = \mathbf{H} \mathbf{x}$, where

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} +\cos \theta & -\sin \theta \\ +\sin \theta & +\cos \theta \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}}.$$

The function \mathbf{H} rotates the two-dimensional vector \mathbf{x} by an angle θ . If $\theta > 0$, the rotation is counter-clockwise; if $\theta < 0$, the rotation is clockwise.

Without using any knowledge of linear algebra, and without any complicated mathematical manipulation, determine the inverse function \mathbf{H}^{-1} ; explain your thought process. Succinctly, but clearly and convincingly, explain whether \mathbf{H} is one-to-one, onto, neither, or both.

Q1.3 (10 Points) Assume to be true that: "At least one EECS student is overworked"; *and* "Every overworked person is tired"; *and* "No Journalism student is overworked." For each independent assertion (a) and (b), select the strongest correct characterization from among the following choices. "The assertion must be true"; "The assertion could be true, but does not necessarily have to be true"; or "The assertion cannot be true." Explain your reasoning succinctly, but clearly and convincingly.

(a) At least one tired person must be an EECS student.

(b) No Journalism student is tired.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.