EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 2
Department of Electrical Engineering and Computer Sciences 19 October 2006
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Philter FIRST Name Kauzal

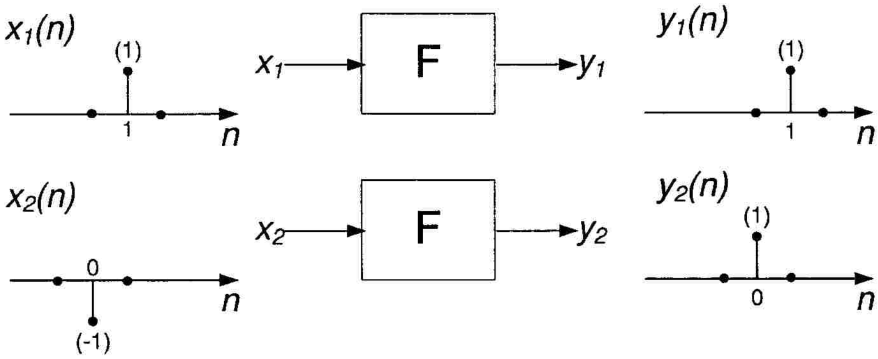
Lab Time M 4-7am

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2	10	10
3	10	10
Total	**45**	45

Q2.1 (20 Points) Consider a discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$. It is known that F is *memoryless.*

The input-output signal pairs (x_i, y_i), $i = 1, 2$, shown in the figure below, are behaviors of the system. The signals x_i and y_i are zero outside the regions shown.

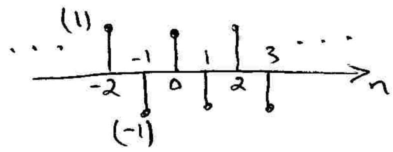


Explain your response to each of the questions below succinctly, but clearly and convincingly.

(a) Provide a well-labeled sketch of the response y of the system, if the input signal x is

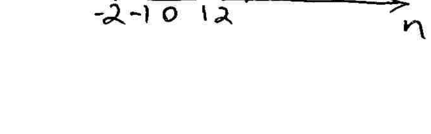
$$\forall n \in \mathbb{Z}, \quad x(n) = e^{i\pi n}.$$

$x(n) = (-1)^n$



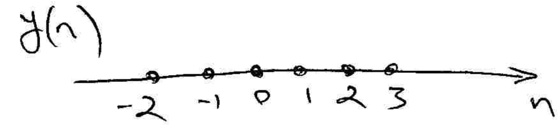
F is memoryless $\Longrightarrow$ There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that $y(n) = f(x(n))$, where (x, y) is a behavior of the system F. Based on the partial information given about F, we know $f(0) = 0$, $f(1) = 1$, $f(-1) = 1$.

The response to x, where $x(n) = e^{i\pi n}$ is then simply $y : \mathbb{Z} \to \mathbb{R}$, $\forall n$, $y(n) = 1$.



(b) Provide a well-labeled sketch of the *zero-input response* of the system; that is, determine the output signal y, if the input signal $x = 0$.

Since $f(0) = 0$, then if $x = 0$, so will y be zero.



Zero input produces zero output. (F has the ZIZO property)

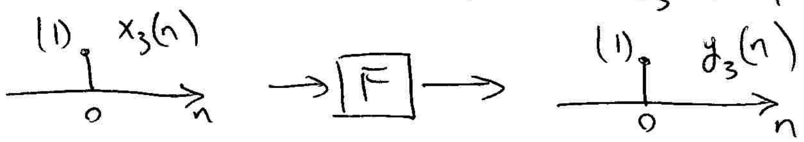
(c) Select the strongest true assertion from the list below. Explain your reasoning succinctly, but clearly and convincingly.

(i) The system must be linear.

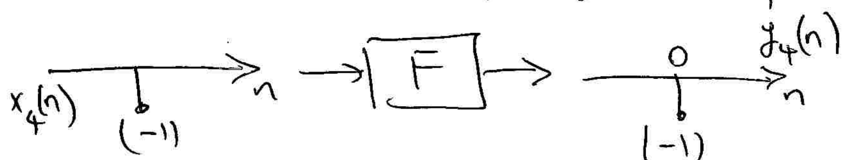
(ii) The system could be linear, but does not have to be.

(iii) The system cannot be linear.

The system is memoryless ⟹ By our definition of memorylessness, it will also be time invariant. Based on the behavior (x_1, y_1), we can conclude that (x_3, y_3) is also a behavior, where $x_3(n) = x_1(n+1)$, $y_3(n) = y_1(n+1)$.



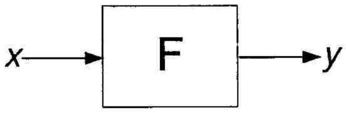
Assume the system is linear. Then $x_4 = -x_3$ must produce $y_4 = -y_3$, i.e.,



Clearly, this contradicts the fact that (x_2, y_2) is a behavior of the system. Hence, F cannot be linear.

Note: This is an example of a system that possesses the ZIZO property, but is not linear.

Q2.2 (10 Points) Consider a continuous-time system $F : [\mathbb{R} \to \mathbb{C}] \to [\mathbb{R} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:



This system complex-conjugates the input signal:

$$y = F(x) = x^*.$$

In other words,

$$\forall t \in \mathbb{R}, \quad y(t) = x^*(t)\,,$$

where * denotes complex conjugation.

Select the strongest true assertion from the list below. Explain your reasoning succinctly, but clearly and convincingly.

(i) The system must be linear.

(ii) The system could be linear, but does not have to be.

(iii) The system cannot be linear.

F satisfies the additivity property, i.e.,
If (x_1, y_1) and (x_2, y_2) are behaviors of F, we know that $y_1 = x_1^*$ and $y_2 = x_2^*$ and $y_1 + y_2 = x_1^* + x_2^* = (x_1 + x_2)^*$, which means $(x_1 + x_2, y_1 + y_2)$ is also a behavior of F.

However, F fails to have the homogeneity property. We show this by a counterexample:

Let $\hat{x} = ix$, then $\hat{y} = \hat{x}^* = -ix^* \neq ix^* = iy$

So (ix, iy) is NOT a behavior of F, even though (x, y) is a behavior.

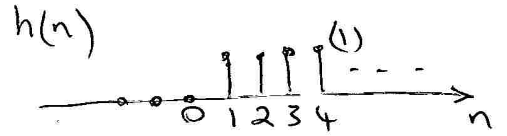
Alternative view: Let $x(t) = a(t) + ib(t)$ where a, b are real-valued signals, representing the real and imaginary components of x. Then $y(t) = x^*(t) = a(t) - ib(t)$. What is $iy(t)$? Ans. $iy(t) = b(t) + ia(t)$
What is $ix(t) = \hat{x}(t)$? Ans. $\hat{x}(t) = a(t) - ib(t)$. Corresponding response $\hat{y}(t) = \hat{x}^*(t) = a(t) + ib(t)$. Clearly, $\hat{y}(t) \neq iy(t)$.

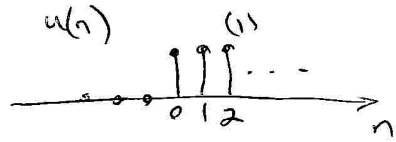
Q2.3 (10 Points) The impulse response $h : \mathbb{Z} \to \mathbb{R}$ of a discrete-time linear, time-invariant (DT-LTI) system $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ is given by:

$$\forall n \in \mathbb{Z}, \quad h(n) = \begin{cases} 0 & n \leq 0 \\ 1 & n \geq 1 . \end{cases}$$

Determine and provide a well-labeled sketch of the response s of the system to the discrete-time unit-step signal u, where the unit-step signal is defined as follows:

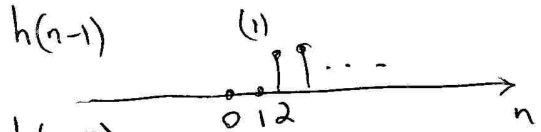
$$\forall n \in \mathbb{Z}, \quad u(n) = \begin{cases} 0 & n \leq -1 \\ 1 & n \geq 0 . \end{cases}$$

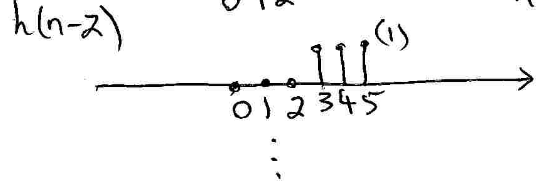




Note: $u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \cdots$

Hence, $s(n) = h(n) + h(n-1) + h(n-2) + \cdots$ (b/c F is LTI)





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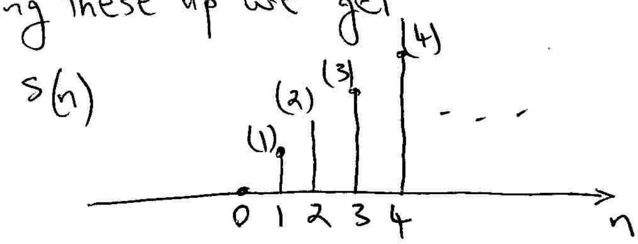
Adding these up we get



$$s(n) = \begin{cases} 0 & n < 0 \\ n & n \geq 0 \end{cases}$$