EECS 20N: Structure and Interpretation of Signals and Systems MIDTERM 2
Department of Electrical Engineering and Computer Sciences 24 October 2006
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ____________________ FIRST Name ____________________

Lab Time ____________________

- **(10 Points)** Print your name and lab time in legible, block lettering above (5 points) AND on the last page (5 points) where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 12.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the twelve numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

Basic Formulas:

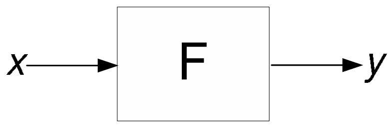
Discrete Fourier Series (DFS) Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n} \ ,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable discrete interval of length p (i.e., an interval containing p contiguous integers). For example, $\sum_{k=\langle p \rangle}$ may denote $\sum_{k=0}^{p-1}$ or $\sum_{k=1}^{p}$.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

MT2.1 (20 Points) Consider a continuous-time system $\mathsf{F} : [\mathbb{R} \to \mathbb{C}] \to [\mathbb{R} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:



This system takes the real part of its input signal:

$$y = F(x) = \mathsf{Re}(x).$$

In other words,

$$\forall t \in \mathbb{R}, \quad y(t) = \mathsf{Re}(x(t)),$$

where $\mathsf{Re}(\cdot)$ denotes taking the real part of a number. For each part below, you must explain your reasoning succinctly, but clearly and convincingly.

(a) Select the strongest true assertion from the list below.

(i) The system must be memoryless.

(ii) The system could be memoryless, but does not have to be.

(iii) The system cannot be memoryless.

(b) Select the strongest true assertion from the list below.

(i) The system must be causal.

(ii) The system could be causal, but does not have to be.

(iii) The system cannot be causal.

(c) Select the strongest true assertion from the list below.

(i) The system must be time invariant.
(ii) The system could be time invariant, but does not have to be.
(iii) The system cannot be time invariant.

(d) Select the strongest true assertion from the list below.

(i) The system must be linear.
(ii) The system could be linear, but does not have to be.
(iii) The system cannot be linear.

MT2.2 (25 Points) The unit-step response[1] s of a discrete-time linear, time-invariant system is given by:

$$\forall n \in \mathbb{Z}, \quad s(n) = (n+1)\,u(n)\,,$$

where u is the unit-step signal characterized as follows:

$$\forall n \in \mathbb{Z}, \quad u(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0\,. \end{cases}$$

Explain your reasoning for each part succinctly, but clearly and convincingly.

(a) Determine and provide a well-labeled sketch of h, the impulse response of the system.

[1] Recall that the unit-step response of a system is, as the name suggests, the response of the system to the unit-step input signal.

(b) Select the strongest true assertion from the list below.

(i) The system must be memoryless.

(ii) The system could be memoryless, but does not have to be.

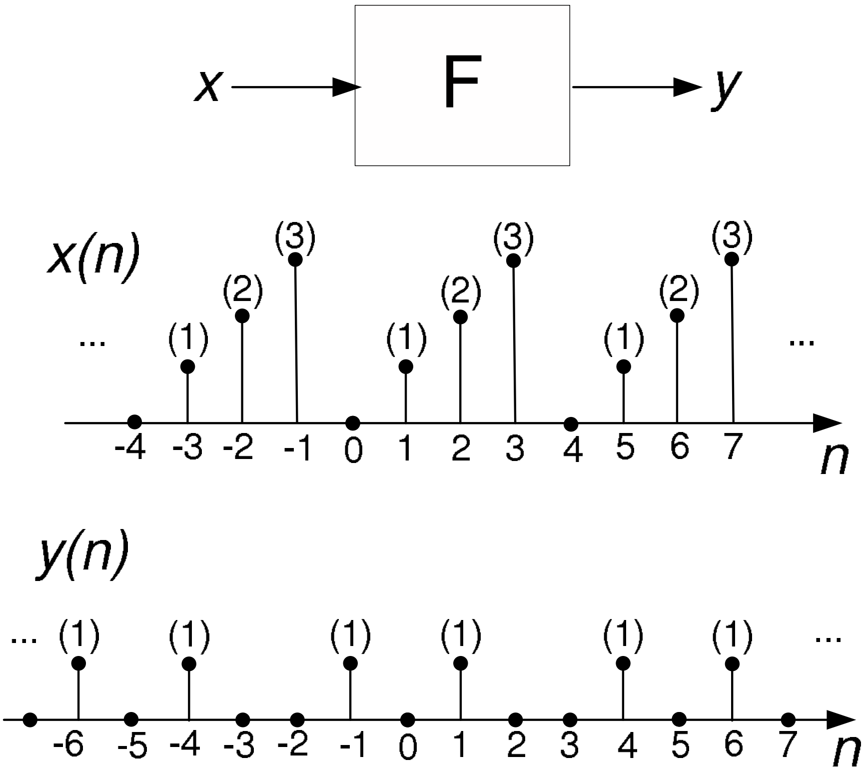
(iii) The system cannot be memoryless.

(c) Determine a simple expression for

$$\sum_{m=-\infty}^{n} h(m)\,.$$

<u>Hint</u>: Your answer will depend on n. You should be able to solve this part even without knowing the impulse response h from part (a).

MT2.3 (25 Points) Consider a discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having a periodic input signal x and a corresponding periodic output signal y, as shown below.



(a) Determine (p_x, ω_x) and (p_y, ω_y), the period and fundamental frequency of x and y, respectively.

(b) Determine the complex exponential discrete Fourier series (DFS) representation of the output signal y. In particular, determine a simple expression for the coefficients Y_k in the DFS expansion

$$Y_k = \frac{1}{p_y} \sum_{n=\langle p_y \rangle} y(n)\, e^{-ik\omega_y n}\,.$$

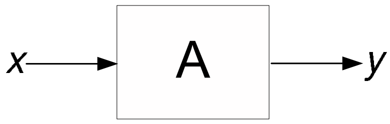
(c) Select the strongest true assertion from the list below. Explain your reasoning succinctly, but clearly and convincingly.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

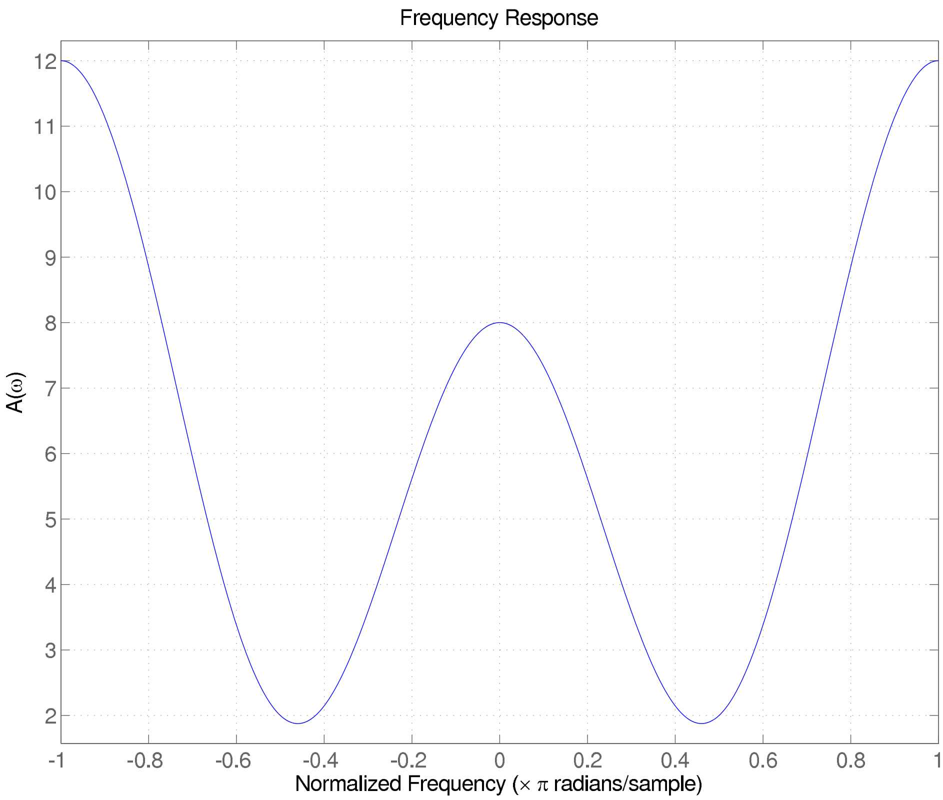
MT2.4 (20 Points) Consider a discrete-time LTI filter A : $[\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having impulse response a and frequency response A. The figure below is a graphical, input-output depiction of the filter:



Recall that the frequency response and impulse response are related as follows:

$$\forall \omega \in \mathbb{R}, \quad A(\omega) = \sum_{n=-\infty}^{\infty} a(n)\, e^{-i\omega n}\,.$$

The figure below depicts $A(\omega), \forall \omega \in [-\pi, +\pi]$. Notice that for this particular filter, $A(\omega)$ is real-valued at all frequencies.



The frequency axis in the figure is normalized by π; hence, for example, the normalized frequencies 0.5 and 1 refer to $\omega = \pi/2$ and $\omega = \pi$ radians per sample, respectively.

Determine a reasonable and simple (possibly approximate) expression for the output y of the filter, if the input x is:

$$\forall n \in \mathbb{Z}, \quad x(n) = e^{i\pi/3} + \cos\left(\frac{4\pi}{5}n\right) + (-1)^n + i^n \, .$$

Note that there is no $''n''$ in the first term. This is not a typographical error.

You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

MT2.5 (15 Points) The impulse response h of a discrete-time LTI system is given by:

$$\forall n \in \mathbb{Z}, \quad h(n) = \left(\frac{1}{2}\right)^n u(n)\,,$$

where u is the unit-step function.

(a) Select the strongest true assertion from the list below.

(i) The system must be causal.

(ii) The system could be causal, but does not have to be.

(iii) The system cannot be causal.

(b) Determine a simple expression for the frequency response H of the system. Recall that the frequency response and impulse response are related as follows:

$$\forall \omega \in \mathbb{R}, \quad H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-i\omega n}\,.$$

<u>Hint</u>: You may find the following helpful. If $|\alpha| < 1$, then $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$.

LAST Name ______________________ FIRST Name ______________________

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Problem	Points	Your Score
Name	10	
1	20	
2	25	
3	25	
4	20	
5	15	
Total	**115**	