EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 3
Department of Electrical Engineering and Computer Sciences 16 November 2006
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Ter FIRST Name Phil

Lab Time 365/24/60/60

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score	Problem	Points	Your Score
Name	5		★	★	★
I	6		IV	7	
II	7		V	7	
III	7		VI	6	
Subtotal	**25**		**Subtotal**	**20**	
TOTAL	★	★	★	**45**	

Q3.1 (40 Points) A causal discrete-time LTI filter $\mathsf{H} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ has an input-output behavior that is at least partially characterized by the linear, constant-coefficient difference equation

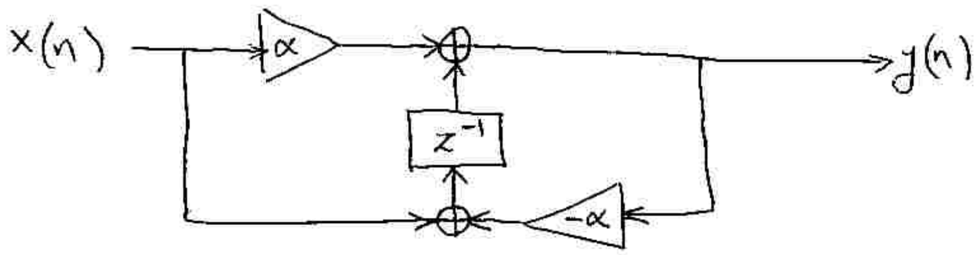
$$y(n) + \alpha\, y(n-1) = \alpha\, x(n) + x(n-1)\,,$$

where the parameter $\alpha \in \mathbb{R}$, $0 < |\alpha| < 1$, is adjustable, but is otherwise constant (with respect to n). Each value of α represents a distinct filter. The input and output signals are denoted by x and y, respectively.

Explain your response to each of the questions below succinctly, but clearly and convincingly.

I (6 Points) Provide a well-labeled delay-adder-gain block diagram representation of H. Your diagram must have the minimal number of storage elements (delays) necessary to represent the filter.

$$y(n) = \alpha\, x(n) + x(n-1) - \alpha\, y(n-1)$$



II **(7 Points)** Determine the filter's impulse response $h : \mathbb{Z} \to \mathbb{R}$. That is, determine an expression for $h(n)$, $\forall n \in \mathbb{Z}$.

$h(n) = \alpha\,\delta(n) + \delta(n-1) - \alpha\, h(n-1)$

System is causal $\Longrightarrow h(n) = 0\,,\ n < 0.$

$h(0) = \alpha\,\delta(0) + \underbrace{\cancel{\delta(-1)}}_{0} - \alpha\, \underbrace{\cancel{h(-1)}}_{0} = \alpha$

$h(1) = \alpha\, \underbrace{\cancel{\delta(1)}}_{0} + \delta(0) - \alpha\, h(0) = 1 - \alpha^2$

$h(2) = \alpha\, \underbrace{\cancel{\delta(2)}}_{0} + \underbrace{\cancel{\delta(1)}}_{0} - \alpha\, h(1) = -\alpha\,(1-\alpha^2)$

$h(3) = -\alpha\, h(2) = (-\alpha)^2 (1-\alpha^2)$

$\vdots$

$$\boxed{h(n) = \begin{cases} 0 & n < 0 \\ \alpha & n = 0 \\ (-\alpha)^{n-1}(1-\alpha^2) & n \geq 1 \end{cases}}$$

III **(7 Points)** Determine the filter's frequency response $H : \mathbb{R} \to \mathbb{C}$. That is, determine an expression for $H(\omega)$, $\forall \omega \in \mathbb{R}$.
You should be able to solve this part independently of your answer to Part (II).

Finding $H(\omega)$ from $h(n)$ using the definition $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n}$ is NOT the best way to go.

Instead, look at the difference equation and note that if $x(n) = e^{i\omega n}$, then $y(n) = H(\omega)\, e^{i\omega n}$; plug these into the equation to get:

$H(\omega) e^{i\omega n} + \alpha\, e^{i\omega(n-1)} H(\omega) = \alpha e^{i\omega n} + e^{i\omega(n-1)}$. Striking out $e^{i\omega n}$ from both sides, and collecting terms, we get:

$(1 + \alpha e^{-i\omega})\, H(\omega) = \alpha + e^{-i\omega} \Longrightarrow H(\omega) = \dfrac{\alpha + e^{-i\omega}}{1 + \alpha e^{-i\omega}}$

IV (7 Points) Determine a simple expression for, and provide a well-labeled sketch of, the filter's magnitude response $|H(\omega)|$, $-\pi \leq \omega < \pi$.

$$H(\omega) = \frac{\alpha + e^{-i\omega}}{1+\alpha e^{-i\omega}} = e^{-i\omega} \frac{1+\alpha e^{i\omega}}{1+\alpha e^{-i\omega}}$$

Note that because $\alpha \in \mathbb{R}$, the numerator $1+\alpha e^{i\omega}$ is the complex conjugate of the denominator $1+\alpha e^{-i\omega}$, i.e.,

$$1+\alpha e^{i\omega} = \left(1+\alpha e^{-i\omega}\right)^* \Longrightarrow \left|1+\alpha e^{i\omega}\right| = \left|1+\alpha e^{-i\omega}\right|$$

Now,

$$|H(\omega)| = \left| e^{-i\omega} \frac{1+\alpha e^{i\omega}}{1+\alpha e^{-i\omega}} \right| = \overset{1}{\cancel{\left|e^{-i\omega}\right|}} \frac{\cancel{\left|1+\alpha e^{i\omega}\right|}^{1}}{\cancel{\left|1+\alpha e^{-i\omega}\right|}} = 1 \Longrightarrow$$

$$|H(\omega)| = 1 \quad \forall \omega$$

Choose one of the descriptions below that best characterizes the filter H:

(a) H is a low-pass filter.

(b) H is a band-pass filter.

(c) H is a high-pass filter.

$\boxed{\text{(d) H is an all-pass filter.}}$

(e) H does *not* belong to any of the filter categories above.

Provide a one-sentence explanation (not exceeding two lines) to support your choice.

The filter has a constant magnitude response $|H(\omega)|$, so it must be allpass.

V (7 Points) Suppose $\alpha = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, and the input signal is characterized by

$$x(n) = \cos\left(\frac{\pi}{2}n\right), \ \forall n \in \mathbb{Z}.$$

Without complicated mathematical manipulations, determine a simple expression for the filter's output sample values $y(n), \forall n \in \mathbb{Z}$.

The corresponding output is $y(n) = |H(\frac{\pi}{2})| \cos\left(\frac{\pi}{2}n + \measuredangle H(\frac{\pi}{2})\right)$.
We know $|H(\frac{\pi}{2})| = 1$, but what is $\measuredangle H(\frac{\pi}{2})$, the phase of the filter's frequency response at frequency $\omega = \pi/2$?

$H(\omega) = e^{-i\omega} \frac{1+\alpha e^{+i\omega}}{1+\alpha e^{-i\omega}}$ complex conjugates $\Rightarrow \measuredangle H(\omega) = \cancel{\measuredangle e^{-i\omega}} + 2\measuredangle(1+\alpha e^{i\omega})$

$\boxed{\measuredangle H(\omega) = -\omega + 2\measuredangle\left(1+\frac{1}{\sqrt{3}}e^{+i\omega}\right)}$

$\measuredangle H(\frac{\pi}{2}) = -\frac{\pi}{2} + 2\measuredangle\left(1+\frac{1}{\sqrt{3}}e^{i\pi/2}\right) = -\frac{\pi}{2} + 2\measuredangle\left(1+\frac{1}{\sqrt{3}}i\right) = \frac{-\pi}{2} + 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow$

$\measuredangle H(\frac{\pi}{2}) = \frac{-\pi}{2} + 2\left(\frac{\pi}{6}\right) = \frac{-\pi}{2} + \frac{\pi}{3} = \frac{-\pi}{6} \Rightarrow y(n) = \cos\left(\frac{\pi}{2}n - \frac{\pi}{6}\right)$

VI (6 Points) Consider each of the following two limiting cases:

(a) $\alpha \to 0$.

(b) $\alpha \to 1$.

For each case, provide a reasonable limiting approximation to the impulse response h and the frequency response H. Briefly explain, for each limiting case, what the filter does to the input signal x.

$H(\omega) = \frac{\alpha + e^{-i\omega}}{1+\alpha e^{-i\omega}} \xrightarrow{\alpha\to 0} H(\omega) \xrightarrow{\alpha\to 0} \frac{e^{-i\omega}}{1} = e^{-i\omega} \Longrightarrow$

As $\alpha \to 0$, $H(\omega) \to e^{-i\omega}$, $h(n) \to \delta(n-1)$ Simple Delay

$H(\omega) \xrightarrow{\alpha\to 1} \frac{1+e^{-i\omega}}{1+e^{-i\omega}} = 1$

As $\alpha \to 1$, $H(\omega) \to 1$ $h(n) \to \delta(n)$ Identity System

You can also see these by taking the limits on the two sides of the difference equation. For example, as $\alpha \to 0$, the LCCDE looks like $y(n) = x(n-1)$.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.