EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 1
Department of Electrical Engineering and Computer Sciences 13 September 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Viktor FIRST Name Komplix

Lab Time 365/52/7/24/60/60/...

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

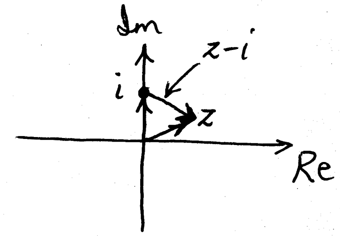
Problem	Points	Your Score
Name	5	5
1	20	20
2	20	20
Total	**45**	45

Q1.1 (20 Points) For each set defined below, provide a well-labeled diagram identifying all the points on the complex plane that belong to it.

The symbol $\mathbb{C}$ denotes the set of complex numbers. To receive full credit, you must explain your reasoning succinctly, but clearly and convincingly. You may tackle each part independently.

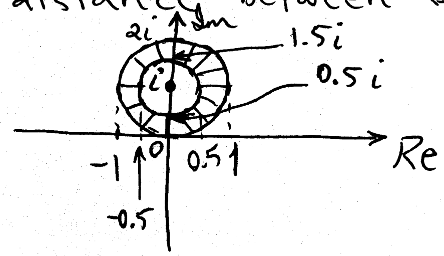
(a) (10 Points)

$$A = \{z \in \mathbb{C} \mid 0.5 \le |z - i| \le 1\}.$$



$z-i$ is the vector from i to z.
$|z-i|$ is the magnitude of the vector from i to z.

The set A consists of every point z in the complex plane that is no closer to i than 0.5 and no farther from it than 1. That is, every element of A is a point on the complex plane at a radial distance between 0.5 and 1 from i.

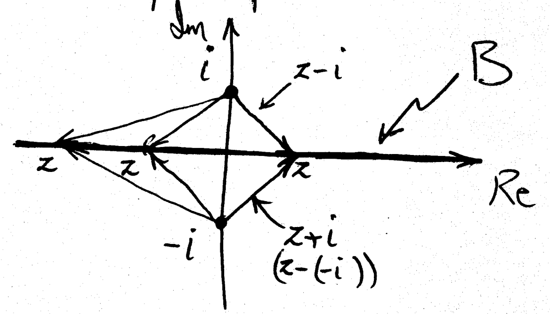


A is the shaded region (boundaries of the annular region included)

(b) (10 Points)

$$B = \{z \in \mathbb{C} \mid |z - i| = |z + i|\}.$$

B is the set of all points on the complex plane that are equidistant from i and $-i$. Clearly, $B = \mathbb{R}$, the set of real numbers (the Re axis on the complex plane shown below).



Q1.2 (20 Points) Consider the quartic (fourth-order) equation

$$z^4 - 4z^2 + 16 = 0.$$

(a) (16 Points) Determine the four solutions (roots) of the equation, and express each root in both a simple rectangular and a simple polar form. Explain your work succinctly, but clearly and convincingly.

Your trigonometry teacher rightfully used to insist that $\cos(\pi/3) = \sin(\pi/6) = 1/2$ and $\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$.

Solve in two steps: (I) Let $x = z^2$ and convert the quartic equation into a quadratic one, which produces two roots x_1, x_2; and (II) Take the square root of each of x_1 and x_2 to arrive at all the four roots of the quartic equation.

(I) $x = z^2 \Rightarrow x^2 - 4x + 16 = 0 \Rightarrow x = 2 \pm \sqrt{4-16} = 2 \pm 2\sqrt{3}\, i \Rightarrow$

$x_1 = 2 + 2\sqrt{3}\, i = 16 e^{i \tan^{-1}\frac{2\sqrt{3}}{2}} = 4 e^{i \tan^{-1}\sqrt{3}} = 4 e^{i\pi/3} \Rightarrow x_1 = 2 + 2\sqrt{3}\, i = 4 e^{i\pi/3}$

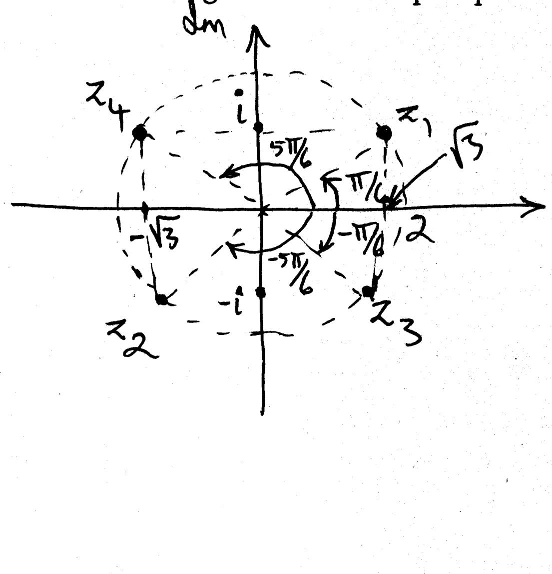
Similarly, $x_2 = 2 - 2\sqrt{3}\, i = 4 e^{-i\pi/3}$

(II) $z^2 = x \Rightarrow z = x^{1/2}$.

Apply the $2\pi k$ phase ambiguity ($k \in \mathbb{Z}$) to x_1 and x_2, and find the square roots by evaluating at $k = 0, 1$. $x_1 = 4 e^{i(\frac{\pi}{3} + 2\pi k)} \Rightarrow x_1^{1/2} = 2 e^{i(\frac{\pi}{6} + k\pi)}$

Similarly, $x_2^{1/2} = 2 e^{i(-\frac{\pi}{6} + k\pi)}$. The four roots are: $z_1 = 2e^{i\pi/6}$ $z_2 = 2e^{i\frac{7\pi}{6}} = 2e^{-i\frac{5\pi}{6}}$ $z_3 = 2e^{-i\pi/6}$ $z_4 = 2e^{i5\pi/6}$

(b) (4 Points) Identify the four solutions as points on a single, well-labeled diagram of the complex plane.



$z_1 = \sqrt{3} + i = 2e^{i\pi/6}$

$z_2 = -\sqrt{3} - i = 2e^{i\frac{7\pi}{6}} = 2e^{-i\frac{5\pi}{6}}$

$z_3 = \sqrt{3} - i = 2e^{-i\pi/6}$

$z_4 = -\sqrt{3} + i = 2e^{i5\pi/6}$

All four roots sit on the circle of radius 2, centered at the origin.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.