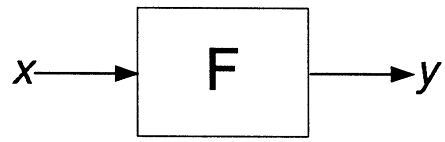
EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 2
Department of Electrical Engineering and Computer Sciences 04 October 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Frequencies FIRST Name ν
Lab Time Dusk

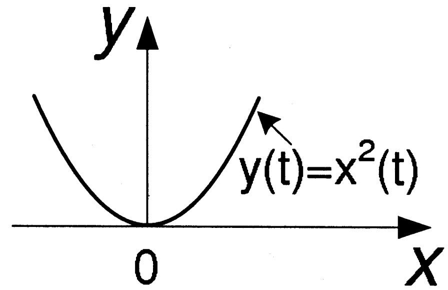
- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	25	25
2	15	15
Total	**45**	45

Q2.1 (25 Points) Consider a continuous-time system $\mathsf{F} : [\mathbb{R} \to \mathbb{R}] \to [\mathbb{R} \to \mathbb{R}]$ having input signal x and output signal y:



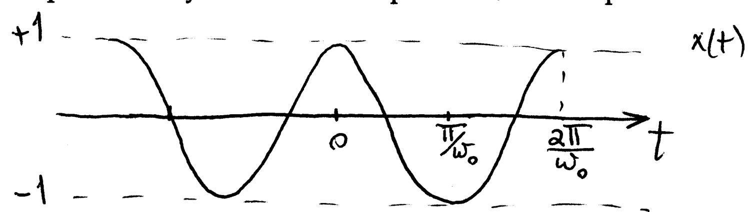
The system is a *square-law device* having the input-output characteristics shown below:



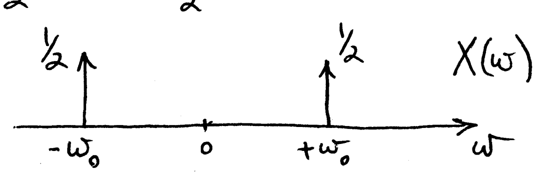
The figure shows that $y(t) = x^2(t), \forall t \in \mathbb{R}$.

Suppose the input to the system is the sinusoid $x(t) = \cos \omega_0 t, \forall t$, where $\omega_0 > 0$.

(a) (9 Points) Provide well-labeled sketches of $x(t)$ and $X(\omega)$, the time-domain signal values and the spectrum of the input signal, respectively. You must explain how you obtain the spectrum; a mere plot will not suffice.



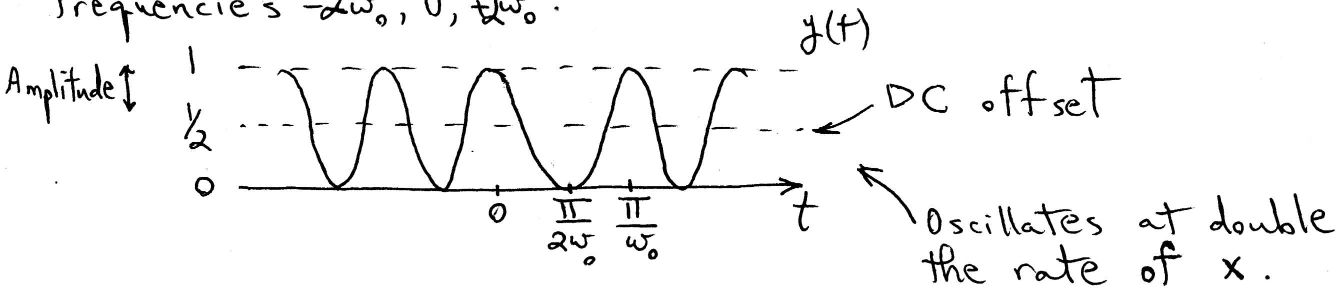
$x(t) = \cos \omega_0 t = \frac{1}{2} e^{i\omega_0 t} + \frac{1}{2} e^{-i\omega_0 t}$ Two frequencies: ω_0 and $-\omega_0$



(b) (9 Points) Determine a simple expression for, and provide a well-labeled sketch of, the output signal $y(t)$.[1] In plain English, articulate how the sketch of $y(t)$ differs from that of $x(t)$.

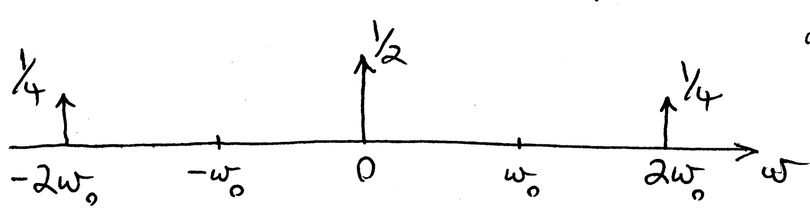
$$y(t) = \cos^2 \omega_0 t = \frac{1}{2} + \frac{1}{2}\cos 2\omega_0 t$$

This is a signal with a DC offset of $\frac{1}{2}$, amplitude $\frac{1}{2}$, and frequencies $-2\omega_0$, 0, $+2\omega_0$.



(c) (7 Points) Provide a well-labeled sketch of $Y(\omega)$, the spectrum of the output signal. Explain the appearance, in the output signal, of any frequency not present in the input signal. Be brief but convincing.

$$y(t) = \frac{1}{2}e^{i0t} + \frac{1}{4}e^{i2\omega_0 t} + \frac{1}{4}e^{-i2\omega_0 t} = \left(\frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2}\right)^2$$



also

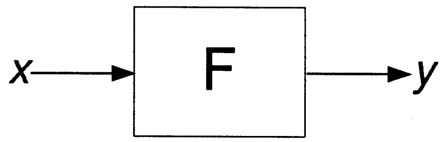
$$= \frac{1}{2} + \frac{1}{2}\cos 2\omega_0 t$$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{e^{i2\omega_0 t} + e^{-i2\omega_0 t}}{2}\right)$$

All these frequencies are new, and they result from the squaring of the input signal, which is a nonlinear operation. In particular, the squaring causes $y(t)$ to be nonnegative for all time, thereby shifting its average value up (to ½), in contrast to $x(t)$, which had an average value of zero (hence, no content @ $\omega = 0$).

[1] You may find the following trigonometric identity helpful: $\cos^2 \alpha = \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$.

Q2.2 (15 Points) Consider the continuous-time system F shown below:



The system F is known to be *time invariant*. Moreover, suppose the input signal x is periodic with fundamental period p_x; that is, $x(t) = x(t + p_x), \forall t$, where $p_x > 0$.

Select the *strongest correct* assertion from the following choices. Explain your reasoning succinctly, but clearly and convincingly.

(I) The output signal y *must* be periodic, and its *fundamental period* p_y *must be* equal to the fundamental period of the input signal; that is, $p_y = p_x$.

(II) The output signal y *must* be periodic, and its *fundamental period* p_y is *at most* equal to the fundamental period of the input signal; that is, $p_y \leq p_x$. Provide an example of a system F such that the inequality is strict ($p_y < p_x$).

(III) The output signal y *may or may not* be periodic. If it is periodic, then its *fundamental period* p_y *must be* equal to the fundamental period of the input signal; that is, $p_y = p_x$.

(IV) The output signal y *may or may not* be periodic. If it is periodic, then its *fundamental period* p_y is *at most* equal to the fundamental period of the input signal; that is, $p_y \leq p_x$. Provide an example of a system F such that the inequality is strict ($p_y < p_x$).

(V) The output signal y *cannot* be periodic.

Let $\hat{x}(t) = x(t+p_x)$. Then $\hat{y}(t) = y(t+p_x)$ because the system is time invariant. But $x(t) = \hat{x}(t) = x(t+p_x)$, $\forall t$, so it must be that $y(t) = \hat{y}(t) = y(t+p_x)$ $\forall t$; recall that a system is a function, so two identical input signals (x and $\hat{x}$) must produce identical output signals (y and $\hat{y}$).

We have shown that $y(t) = y(t+p_x)$, $\forall t$, which means the output must be periodic, and that it's fundamental period p_y, whatever it is, cannot exceed p_x.

Can $p_y < p_x$? Sure. All we need is one example system and a sample input signal for which this is true. Let the system be a square-law device: $y(t) = x^2(t), \forall t$. The input signal $x(t) = \cos \omega_x t$, which has period $p_x = 2\pi/\omega_x$ will produce $y(t) = \frac{1}{2} + \frac{1}{2}\cos 2\omega_0 t$, which has $p_y = \frac{\pi}{\omega_x} < p_x$.