EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 3
Department of Electrical Engineering and Computer Sciences 08 November 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Eko FIRST Name Mr

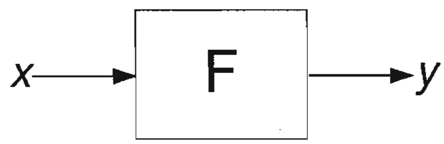
Lab Time Strike of Midnight

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	40	40
Total	**45**	45

Q3.1 (40 Points) Consider a lecture hall whose physical environment produces acoustic distortion in a speaker's sound. What the listener hears is not necessarily the same as what the speaker utters.

A particular lecture hall produces linear, time-invariant acoustic distortion, which can be modeled reasonably well by a DT-LTI system F whose input x denotes the sound of the speaker's voice and whose output y represents the speaker's sound as perceived by a listener.



In our simple model, what the listener hears is the superposition of two component signals: one component is the sound of the speaker's voice arriving via a direct path, without distortion or delay; the second component is the sound of the speaker's voice arriving via an indirect path—as a reflection from a wall, ceiling, or floor—and suffering from both attenuation and delay.

The impulse response of a system F that models the situation described above is characterized by

$$f(n) = \delta(n) + \frac{1}{2}\delta(n-3).$$

(a) (8 Points) Determine a reasonably simple expression for $F(\omega)$, the frequency response of the acoustic model of the lecture-hall environment.

Also provide a well-labeled sketch of the magnitude response $|F(\omega)|$.

Quick Method: Recognize $\delta(n) \longleftrightarrow 1$ & $\delta(n-3) \longleftrightarrow e^{-i3\omega}$

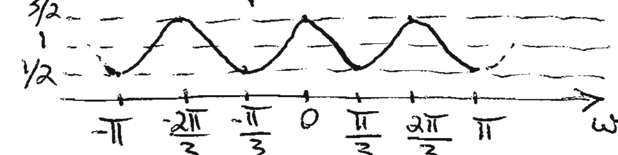
So $f(n) = \delta(n) + \frac{1}{2}\delta(n-3) \longleftrightarrow F(\omega) = 1 + \frac{1}{2}e^{-i3\omega}$

Alternative: $F(\omega) = \sum_{n=-\infty}^{\infty} f(n) e^{-i\omega n} = \sum_{n=-\infty}^{\infty} \left[\delta(n) + \frac{1}{2}\delta(n-3)\right] e^{-i\omega n}$

Only two terms are non-zero: $n=0$ and $n=3 \Rightarrow F(\omega) = 1 + \frac{1}{2}e^{-i3\omega}$

To Find $|F(\omega)|$: $|F(\omega)| = \left|1 + \frac{1}{2}\cos 3\omega - \frac{i}{2}\sin 3\omega\right| = \sqrt{\left(1 + \frac{1}{2}\cos 3\omega\right)^2 + \frac{1}{4}\sin^2 3\omega}$

$|F(\omega)| = \sqrt{\frac{5}{4} + \cos 3\omega}$



You can also sketch $|F(\omega)|$ using the graphical analysis of traversing the unit circle, but it's more nuanced. One way is to analyze $G(\omega) = 1 + \frac{1}{2}e^{-i\omega}$ and the argue that $F(\omega) = G(3\omega)$, so F has a frequency-compressed plot relative to G.

(b) (4 Points) Determine a reasonably simple expression for the linear, constant-coefficient, difference equation that can be a fair representative of the lecture hall's acoustic distortion.

$$y(n) = x(n) + \frac{1}{2}x(n-3)$$

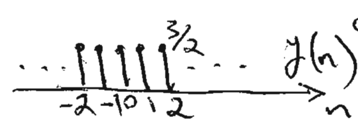
It's when you replace $x(n)$ with $\delta(n)$ in this equation that you get the impulse response f.

(c) (28 Points) Determine the response y of the system (lecture hall) to each of the following prospective input signals?

(i) $x(n) = 1, \forall n \in \mathbb{Z}$. Provide a well-labeled sketch of the response y.

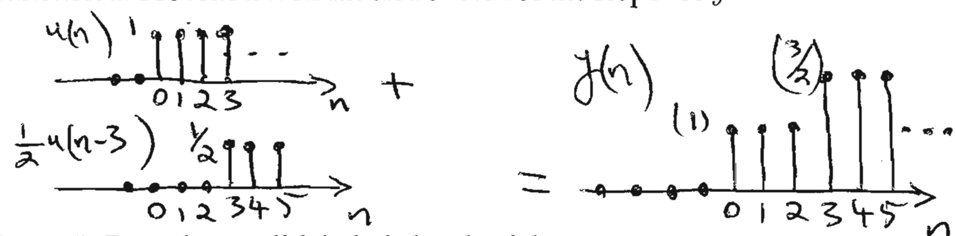
Method 1: $x(n) = e^{i0n} \Longrightarrow y(n) = F(0)e^{i0n}$; we know $F(0) = \frac{3}{2} \Rightarrow y(n) = \frac{3}{2}$ $\forall n$

Method 2: $y(n) = x(n) + \frac{1}{2}x(n-3) = 1 + \frac{1}{2} = \frac{3}{2}, \forall n$



(ii) The unit-step function u. Provide a well-labeled sketch of the response y.

$$y(n) = u(n) + \frac{1}{2}u(n-3)$$

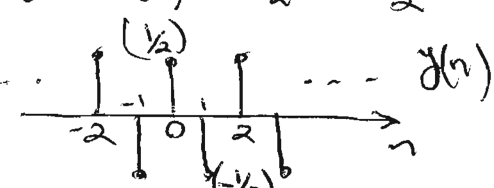


(iii) $x(n) = (-1)^n, \forall n \in \mathbb{Z}$. Provide a well-labeled sketch of the response y.

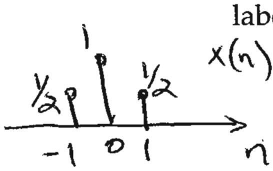
Method 1: $x(n) = e^{i\pi n} \Longrightarrow y(n) = F(\pi)e^{i\pi n}$; we know $F(\pi) = 1 + \frac{1}{2}e^{-i3\pi} = \frac{1}{2} \Rightarrow$

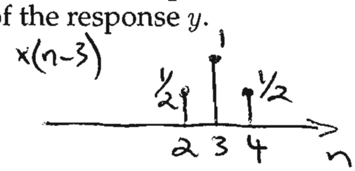
$y(n) = \frac{1}{2}(-1)^n$

Method 2: $y(n) = (-1)^n + \frac{1}{2}(-1)^{n-3} = (1 - \frac{1}{2})(-1)^n = \frac{1}{2}(-1)^n$

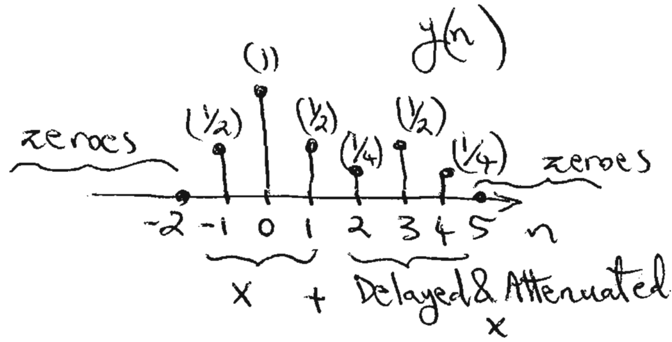


(iv) The triangular signal $x(n) = \frac{1}{2}\delta(n+1) + \delta(n) + \frac{1}{2}\delta(n-1)$. Provide a well-labeled sketch of the response y.





$$y(n) = x(n) + \frac{1}{2}x(n-3)$$



(v) $x(n) = \cos\left(\frac{\pi}{3}n\right)$, $\forall n \in \mathbb{Z}$. Provide a reasonably simple expression for the response y.

$$x(n) = \frac{1}{2}e^{i\frac{\pi}{3}n} + \frac{1}{2}e^{-i\frac{\pi}{3}n} \Longrightarrow y(n) = \frac{1}{2}F\left(\frac{\pi}{3}\right)e^{i\frac{\pi}{3}n} + \frac{1}{2}F\left(-\frac{\pi}{3}\right)e^{-i\frac{\pi}{3}n} \quad \Big\} \Longrightarrow$$

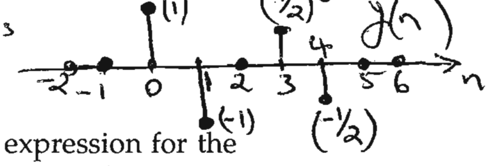
$$F\left(\frac{\pi}{3}\right) = 1 + \frac{1}{2}e^{-i\frac{\pi}{3}\cdot 3} = \frac{1}{2} \qquad F\left(-\frac{\pi}{3}\right) = 1 + \frac{1}{2}e^{i\frac{\pi}{3}\cdot 3} = \frac{1}{2}$$

$$y(n) = \frac{1}{4}e^{i\frac{\pi}{3}n} + \frac{1}{4}e^{-i\frac{\pi}{3}n} = \frac{1}{2}\cos\left(\frac{\pi}{3}n\right) \Longrightarrow y(n) = \frac{1}{2}\cos\left(\frac{\pi}{3}n\right)$$

(vi) $x(n) = \delta(n) - \delta(n-1)$, $\forall n \in \mathbb{Z}$. Provide a well-labeled sketch of the response y.

Method 1: $y(n) = x(n) + \frac{1}{2}x(n-3) = \delta(n) - \delta(n-1) + \frac{1}{2}\delta(n-3) - \frac{1}{2}\delta(n-4)$

Method 2: x picks out the "edges" in f (because x is a discrete-time doublet) $\Longrightarrow$ y(n) is zero except at n=0 (where f jumps up by one unit), n=1 (where f drops by one unit), n=3 (where f jumps by ½ unit), and n=4 (where f drops by ½ unit)



(vii) $x(n) = e^{i\pi n/2}$, $\forall n \in \mathbb{Z}$. Provide a reasonably simple expression for the response y. You may find it useful to know that $\tan(\pi \cdot 0.1476) \approx 0.50$.

$$y(n) = F\left(\frac{\pi}{2}\right)e^{i\frac{\pi}{2}n}$$

$$F\left(\frac{\pi}{2}\right) = 1 + \frac{1}{2}e^{-i\frac{3\pi}{2}} = 1 + \frac{1}{2}i = \sqrt{1 + \frac{1}{4}}\; e^{i\tan^{-1}(1/2)} = \sqrt{5/4}\; e^{i\pi\cdot(0.1476)}$$

$$\Rightarrow y(n) = \frac{\sqrt{5}}{2}\, e^{i\pi\left(\frac{n}{2} + 0.1476\right)}$$