Practice Problems for Midterm #2, Fall 1998.

1. Consider a continuous-time signal x where for all t ∈ Reals,

$$x(t) = \sum_{k=-\infty}^{\infty} r(t-k)$$

where

$$r(t) = \begin{cases} 1 & 0 \le t < 0.5 \\ 0 & \text{otherwise} \end{cases}.$$

Define $Sampler_T : ContSignals \to DiscSignals$ in the usual way to be a sampler with sampling interval T, where if $y = Sampler_T(x)$, then for all integers n, $y(n) = x(nT)$. Define $IdealDiscToCont : DiscSignals \to ContSignals$ to be an ideal reconstruction system.

a) Is $x(t)$ periodic? If so, what is the period?
b) Suppose that $T = 1$. Give a *simple* expression for $y = Sampler_T(x)$.
c) For the same $T = 0.5$, give a *simple* expression for $IdealDiscToCont(Sampler_T(x))$.
d) Find an upper bound for T (in seconds) such that $x = IdealDiscToCont(Sampler_T(x))$, or argue that no value of T makes this assertion true.

2. Consider an LTI discrete-time system Filter with impulse response

$$h(n) = \delta(n) + \delta(n-2)$$

where δ is the Kronecker delta function.

a) Sketch h(n).
b) Suppose x(n) = cos(ω n), whereω = π/2 radians/sample. Give a simple expression for y = Filter(x).
c) Give an expression for $H(\omega)$ that is valid for all ω, where $H = DTFT(h)$.

3. Consider a system $Abs : ContSignals \to ContSignals$ where if $y = Abs(x)$ then

$$y(t) = |x(t)|$$

(the absolute value).

a) Show that this system is not linear.
b) Show that this system is time-invariant.

4. Suppose that the frequency response of a discrete-time LTI system *Filter* is given by

$$H(\omega) = \cos(\omega)$$

where ω has units of radians/sample.

a) Suppose the input is $x(n) = e^{j\pi n}$. Given an expression for the output $y = Filter(x)$.
b) Find $h(n)$, the impulse response.
c) Is *Filter* causal?