More Practice Problems for Midterm #2, Fall 1998.

1. Consider a continuous-time LTI system *H*. Suppose that when the input is given by

$$x(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

then the output is given by

$$y(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \\ \sin(\pi(t-1)) & 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

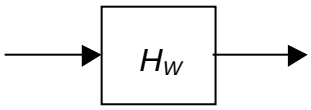
for all $t \in$ *Reals*.

a) Carefully sketch these two signals.
b) Give an expression and a sketch for the output of the same system if the input is

$$x'(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \\ -\sin(\pi(t-1)) & 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

2. Suppose you are given the following building blocks:
   - An LTI system that is an ideal continuous-time lowpass filter with frequency response

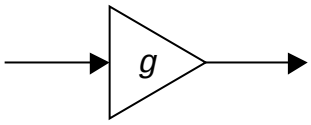
$$H(\omega) = \begin{cases} 1 & -W < \omega < W \\ 0 & \text{otherwise} \end{cases}$$



     where *W* is a parameter you can set.

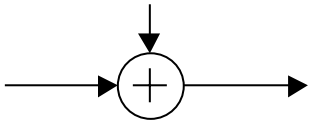
   - A gain block, where if the input is *x* then the output is given by

$$y(t) = gx(t)$$

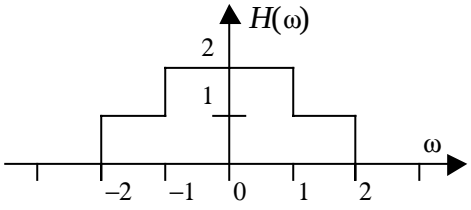


     for all $t \in$ *Reals*, where again *g* is a parameter you can set.

   - An adder, which can add two continuous time signals.

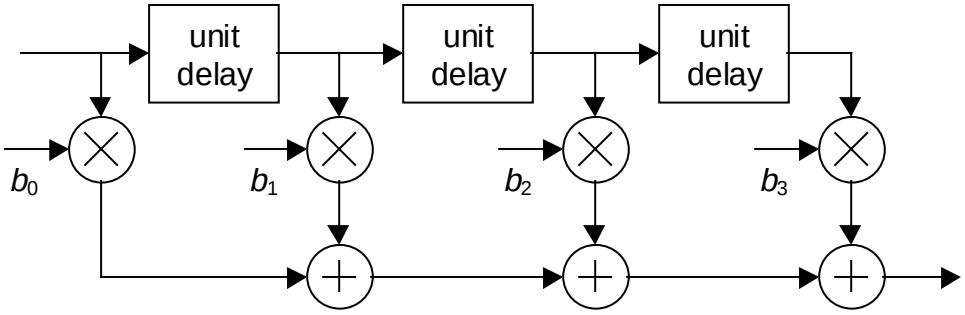


Use these building blocks to construct a system with the frequency response shown below:



3. Consider a continuous-time signal x with Fourier transform X. Find expressions for the Fourier transform of the following signals in terms of the Fourier transform X.
   a) y such that $\forall\ t \in$ *Reals*, $y(t) = x(at)$, for some real number a.
   b) w such that $\forall\ t \in$ *Reals*, $w(t) = x(t)\, e^{j\alpha t}$, for some real number α.
   c) z such that $\forall\ t \in$ *Reals*, $z(t) = x(t)\cos(\alpha t)$, for some real number α.

4. Consider the FIR system described by the following block diagram:



   Suppose that this system has frequency response $H(\omega)$. Define a new system with the identical structure as above, except that each unit delay is replaced by a double delay (two cascaded unit delays). Find the frequency response of that system in terms of $H(\omega)$.