Practice Problems for Final Exam Fall 1998.

1. Consider a continuous-time signal x where for all $t \in Reals$,

$$x(t) = \begin{cases} 1 & 0 \le t < 0.5 \\ 0 & \text{otherwise} \end{cases}.$$

   Define $Sampler_T : ContSignals \to DiscSignals$ in the usual way to be a sampler with sampling interval T, where if $y = Sampler_T\,(x)$, then for all integers n, $y(n) = x(nT)$. Define $IdealDiscToCont : DiscSignals \to ContSignals$ to be an ideal reconstruction system.

   a) Is $x(t)$ periodic? If so, what is the period?
   b) Suppose that $T = 1$. Find $y = Sampler_T(x)$.
   c) Find an upper bound for T (in seconds) such that $x = IdealDiscToCont\,(\,Sampler_T(x))$, or argue that no value of T makes this assertion true.

2. Consider an LTI discrete-time system *Filter* with impulse response

$$h(n) = \delta(n) - \delta(n-1)$$

   where δ is the Kronecker delta function.

   a) Sketch $h(n)$.
   b) Consider the discrete-time signal x given by $x(n) = 1\ \forall\ n \in Ints$. Find $y = Filter(x)$.
   c) Give an expression for $H(\omega)$ that is valid for all ω, where $H = DTFT(h)$.

3. Consider a continuous-time system $D : ContSignals \to ContSignals$ where if $y = D(x)$ then $\forall\ t \in Reals$

$$y(t) = x(t-1).$$

   a) Is D linear? Justify your answer.
   b) Is D time-invariant? Justify your answer.

4. Consider a continuous-time system $TimeScale : ContSignals \to ContSignals$ where if $y = TimeScale(x)$ then $\forall\ t \in Reals$

$$y(t) = x(2t).$$

   a) Is *TimeScale* linear? Justify your answer.
   b) Is *TimeScale* time-invariant? Justify your answer.

5. Suppose that the frequency response of a discrete-time LTI system *Filter* is given by

$$H(\omega) = |\sin(\omega)|$$

   where ω has units of radians/sample. Suppose the input is the discrete-time signal x given by $x(n) = 1\ \forall\ n \in Ints$. Give a simple expression for $y = Filter(x)$.

6. Consider a continuous-time periodic signal x with fundamental frequency $\omega_0 = 1$ radian/second. Recall that the Fourier series in general is written as

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k)$$

   Suppose that for this particular example,

$$A_k = \begin{cases} 1 & k = 0,\ 1,\ \text{or } 2 \\ 0 & \text{otherwise} \end{cases}$$

   a) Recall that the Fourier series can be written in terms of complex exponentials as follows:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}$$

Find the coefficients X_k for all $k \in$ *Ints*.

b) Consider a continuous-time LTI system *Filter* : *ContSignals* $\to$ *ContSignals* with frequency response

$$H(\omega) = \cos(\pi\omega / 2)$$

Find y = *Filter*(x). I.e., give a simple expression for $y(t)$ that is valid for all $t \in$ *Reals*.

c) For y calculated in (b), find the fundamental frequency in radians per second. I.e., find the largest $\omega_0' > 0$ such that $\forall\ t \in$ *Reals* and $\forall\ k \in$ *Ints*,

$$y(t) = y(t + k2 / \omega_0').$$