Practice Problems

11/23/98

State machines

1. Construct a state machine with input and output set $U = Y = \{0,1\}$, such that if $u(0), u(1), \cdots$ is any input sequence, the output sequence is

$$y(n) = \begin{cases} 1, & \text{if } (u(n-3), u(n-2), u(n-1)) = (1,1,1) \\ 0, & \text{otherwise} \end{cases}$$

   In words: the machine outputs 1 if the three previous inputs are all 1's, otherwise it outputs 0.

2. Construct a state machine with the same input and output set as above such that the machine outputs 0 if the three previous inputs are either 111 or 101, otherwise it outputs 1.

3. Take the state machine constructed above. Enclose it in a block with a single inport and a single outport. Take the symbol set to be $S = \{0,1\}$. Bind the inputs, outputs, and transitions of the internal state machine to the port symbols.

4. Take the enclosed state machine, and now connect the outport and inport. Then there is no external input and the machine produces a single state trajectory. What is this trajectory?

5. Consider the interconnected machines of Figure 1. Describe the joint state machine by a transition diagram.

6. A non-deterministic machine generates several possible state trajectories and output sequences for a given input sequence. The relation between input sequences and output sequences is given as a relation

$$H \subset InputSequences \times OutputSequences$$

   where $(u, y) \in H$ if y is a possible output sequence corresponding to u. For the state machine of Figure 2 write down the state transition function and igure out the relation H. Remember the self-loops in each state.

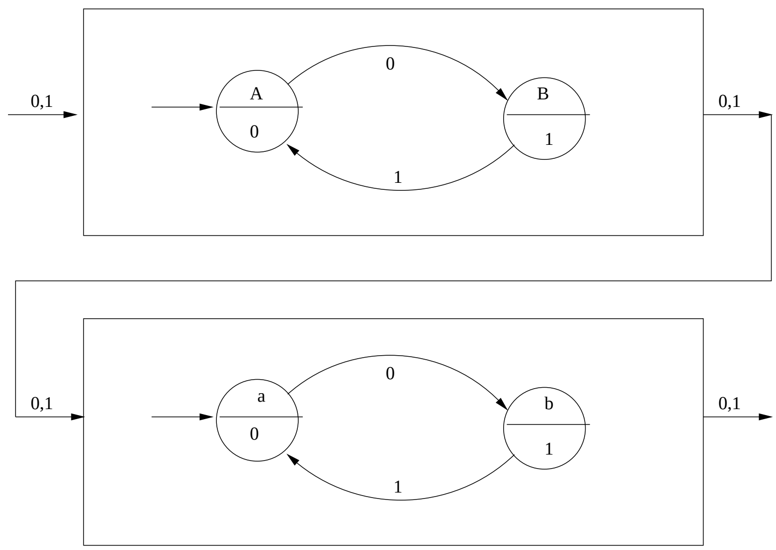

Figure 1: Interconnected machines

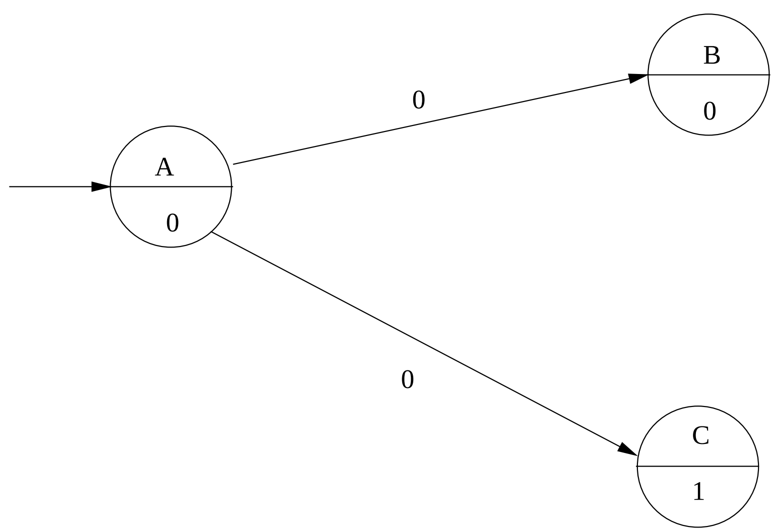

Figure 2: A nondeterministic machine

7. Figure 3 shows a state machine M with 4 states. Find a non-deterministic machine N with 3 states which simulates M such that between two successive outputs 0 N outputs atleast one 1. You are free to choose the input symbols.

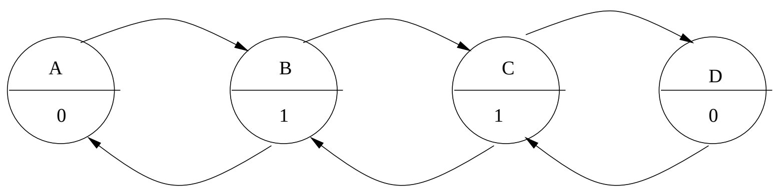

Figure 3: The machine M

Difference equation systems

1. Construct a difference equation system whose zero-state output response is given by

$$y(t) = u(t-1) + u(t-2)$$

2. The A matrix of a two-dimensional system is

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Calculate the zero-input state trajectory $x(0), x(1), x(2), \cdots$ if

(a) the initial state is (1,0),

(b) the initial state is (0,1),

(c) the initial state is (1,1).

3. A one-dimensional system is described by: $\forall t \geq 0$,

$$\begin{aligned} x(t+1) &= x(t) + u(t) \\ y(t) &= x(t) \end{aligned}$$

Suppose the initial state is $x_0 = a$. Find $u*$ so that if the first three inputs are $u(0) = u(1) = u(2) = u*$, then $y(3) = 0$.

4. A single-input, two-dimensional system has the A matrix given by:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

and the b vector is $b = [0, 1]'$. Suppose the initial state x_0 is zero. Find the input sequence $u(0), u(1)$ so that the state at step 2 is $x(2) = (1, 2)$.

5. Suppose the A matrix of a two-dimensional system is

$$A = \sigma \begin{bmatrix} \cos(\pi/6) & \sin(\pi/6) \\ -\sin(\pi/6) & \cos(\pi/6) \end{bmatrix}$$

Suppose the initial state $x_0 = (1, 0)$, and the input is zero. Sketch the state trajectory for $t = 0, 1, \cdots, 12$ for the cases:

(a) $\sigma = 0$

(b) $\sigma = 0.9$

(c) $\sigma = 1.1$.

Sets, functions, predicates

1. A relation F from a set X to a set Y is a subset $F \subset X \times Y$. Figure 4 shows a graph consisting of four nodes $X = \{a, b, c, d\}$. There are edges connecting the nodes with labels 0 or 1 or both. Construct the following relations on $X \times X$

$$\begin{aligned} F_0 &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 0 \} \\ F_1 &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 1 \} \\ F_{01} &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 0 \text{ and an edge labeled } 1 \} \\ F_{0or1} &= \{(x, y) \mid \text{ there is an edge from } x \text{ to } y \text{ labeled } 0 \text{ or} 1 \} \end{aligned}$$

(a) Write each of these four relations as a list.

(b) Are the following assertions true or false?

$$F_{01} = F_0 \cap F_1, \;\; F_{0or1} = F_0 \cup F_1$$

2. Take the same graph as in Figure 4. Define the relation

$$F_{00} = \{(x, y) \mid \text{ two consecutive edges labeled } 0 \text{ connect } x \text{ to } y\}$$

List F_{00}.

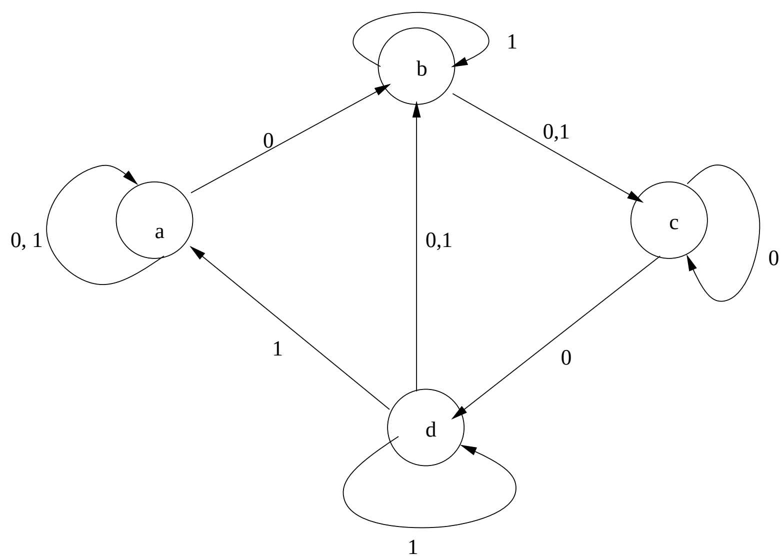

Figure 4: A graph