EECS 20. Final Exam
May 15, 2000.

Please use these sheets for your answer. Use the backs if necessary. **Write clearly and show your work.**

Print your name and lab time below

Name: ____________________________________

Lab time: ________________________________

Problem 1 (9):

Problem 2 (24):

Problem 3 (12):

Problem 4 (28):

Problem 5 (8):

Problem 6 (10):

Problem 7 (9):

Total:

1. **9 points** Consider the continuous-time systems S_k given by, $\forall\, t \in$ *Reals*,

$$
\begin{aligned}
(S_1(x))(t) &= x(t-2), \\
(S_2(x))(t) &= x(t+2), \\
(S_3(x))(t) &= x(t)-2, \\
(S_4(x))(t) &= x(2-t), \\
(S_5(x))(t) &= x(2t), \\
(S_6(x))(t) &= t^2 x(t),
\end{aligned}
$$

(a) Which of these systems is linear?

(b) Which of these systems is time invariant?

(c) Which of these systems is causal?

2. **24 points.** Suppose that the following difference equation relates the input x and output y of a discrete-time, causal LTI system S,

$$y(n) + \alpha y(n-1) = x(n) + x(n-1),$$

for some constant α.

(a) Find the impulse response h.

(b) Find the frequency response H.

(c) Find a sinusoidal input with non-zero amplitude such that the output is zero.

(d) Give Matlab statements to create a reasonable plot of the magnitude of the frequency response. Assume α is given by a Matlab variable `alpha`.

(e) Find a state-space description for this system (define the state s and find A, b, c^T, d).

(f) Suppose $\alpha = 1$. Find the impulse response and frequency response. Make sure your answer makes sense (check it against the original difference equation).

3. **12 points.** Each of the statements below refers to a discrete-time system S with input x and output y. Determine whether the statement is true or false. **NOTE:** No partial credit will be given, so consider your answer carefully. The signal $u(n)$ used below is the **unit step** defined by

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The signal δ used below is the Kronecker delta function.

(a) Suppose you know that if x is a sinusoid then y is a sinusoid. Then you can conclude that S is LTI.

(b) Suppose you know that S is LTI, and that if $x(n) = \cos(\pi n/2)$, then $y(n) = 2\cos(\pi n/2)$. Then you have enough information to determine the frequency response.

(c) Suppose you know that S is LTI, and that if $x(n) = \delta(n)$, then $y(n) = (0.9)^n u(n)$. Then you have enough information to determine the frequency response.

(d) Suppose you know that S is LTI, and that if $x(n) = u(n)$, then $y(n) = (0.9)^n u(n)$. Then you have enough information to determine the frequency response.

(e) Suppose you know that S is causal, and that input $x(n) = \delta(n)$ produces output $y(n) = \delta(n) + \delta(n-1)$, and input $x'(n) = \delta(n-2)$ produces output $y'(n) = 2\delta(n-2) + \delta(n-3)$. Then you can conclude that S is not LTI.

(f) Suppose you know that S is causal, and that if $x(n) = \delta(n) + \delta(n-2)$ then $y(n) = \delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3)$. Then you can conclude that S is not LTI.

4. **28 points** Consider the continuous-time signal

$$x(t) = \cos(10\pi t) + \cos(20\pi t) + \cos(30\pi t).$$

(a) Find the fundamental frequency. Give the units.

(b) Find the Fourier series coefficients $A_0, A_1, \cdots$ and $\phi_1, \phi_2, \cdots$ in

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k)$$

(c) Let y be the result of sampling this signal with sampling frequency 10 Hz. Find the fundamental frequency for y, and give the units.

(d) For the same y, find the discrete-time Fourier series coefficients, $A_0, A_1, \cdots$ and $\phi_1, \cdots$ in

$$y(n) = A_0 + \sum_{k=1}^{K} A_k \cos(k\omega_0 n + \phi_k)$$

where

$$K = \begin{cases} (p-1)/2 & \text{if } p \text{ is odd} \\ p/2 & \text{if } p \text{ is even} \end{cases}$$

where p is the period.

(e) Find

$$w = \mathit{IdealInterpolator}_T(\mathit{Sampler}_T(x))$$

for $T = 0.1$ seconds.

(f) Is there any aliasing distortion caused by sampling at 10 Hz? If there is, describe the aliasing distortion in words.

(g) Give the smallest sampling frequency that avoids aliasing distortion.

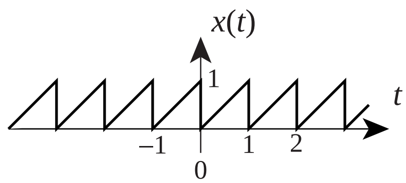


Figure 1: A sawtooth signal

5. **8 points** Consider the sawtooth signal shown in figure 1. This is a periodic, continuous-time signal. Suppose it is filtered by an LTI system with frequency response

$$H(\omega) = \begin{cases} 1 & \text{if } |\omega| \leq 2.5 \text{ radians/second} \\ 0 & \text{otherwise} \end{cases}$$

What is the output?

6. **10 points** Determine for each of the following statements whether it is true or false. **NOTE:** No partial credit will be given, so consider your answer carefully.

   (a) Suppose that state machine A simulates state machine B. Then B must also simulate A.

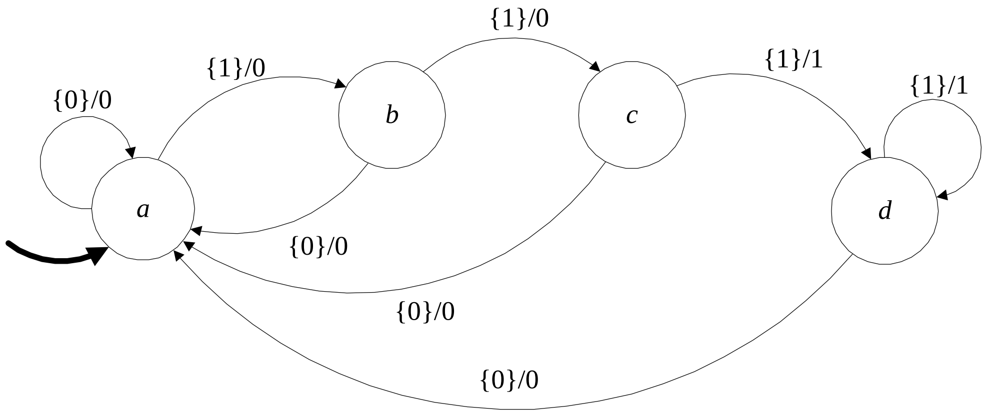
   (b) Suppose that deterministic state machine A simulates deterministic state machine B. Then B must also simulate A.

   (c) Consider two state space models for LTI systems that are distinct (at least one of A, b, c, d are different), but have the same zero-state impulse response. The state machines they describe are bisimilar.

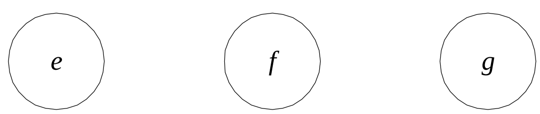
   (d) Suppose two state space models for LTI systems are bisimilar. Then their frequency response is the same.

   (e) A **behavior** of a state machine is a pair of sequences (x, y), where if x is an input to the state machine, then y is an output. The set of behaviors is the set of all possible such pairs for a given machine. If two machines have identical sets of behaviors, then they are bisimilar.

7. **9 points** Consider the following state machine:



which has alphabets $Inputs = Outputs = \{0, 1, absent\}$. Find a three state machine that is bisimilar to this one, and give the bisimulation relation. Give your state machine by adding arcs and careful labels to the following diagram:



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