EECS 20. Midterm No. 1
February 23, 2000.

Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab time below

Name: _____

Lab time: _____

Problem 1:

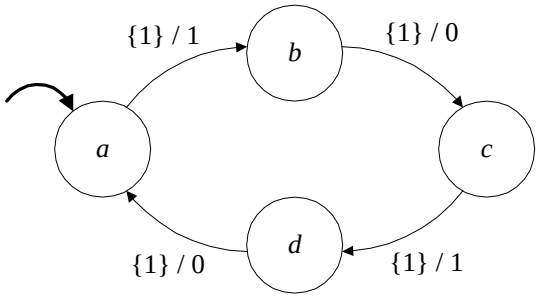
Problem 2:

Problem 3:

Problem 4:

Total:

1. **40 points.** Consider the state machine below:



where

$$\textit{Inputs} = \{1, \textit{absent}\} \quad \text{and} \quad \textit{Outputs} = \{0, 1, \textit{absent}\}$$

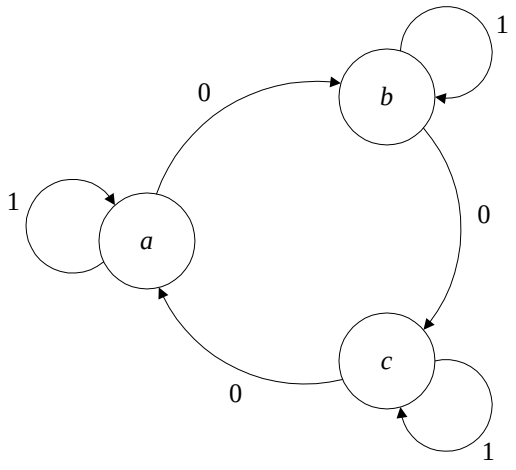
(a) Is this machine deterministic or nondeterministic?

(b) Give the update table.

(c) Find a deterministic state machine that is bisimilar to this one and has only two states Give it as a state transition diagram by completing the diagram below:

(d) Give the bisimulation relation.

2. **30 points.** Let $X = \{a, b, c\}$ represent a set of circles in the following picture:



Consider the following relations, all subsets of $X \times X$:

$$
\begin{aligned}
F_0 &= \{(x_1, x_2) \mid \text{there is an arc going from } x_1 \text{ to } x_2 \text{ with a 0}\} \\
F_1 &= \{(x_1, x_2) \mid \text{there is an arc going from } x_1 \text{ to } x_2 \text{ with a 1}\} \\
F_{0and1} &= \{(x_1, x_2) \mid \text{there are two arcs going from } x_1 \text{ to } x_2\text{, one with a 0 and one with a 1}\} \\
F_{0or1} &= \{(x_1, x_2) \mid \text{there is an arc going from } x_1 \text{ to } x_2 \text{ with a 0 or one with a 1}\}
\end{aligned}
$$

(a) Give the elements of the four relations.

(b) Which of the four relations are the graph of a function of the form $f\colon X \to X$? List **all** that are such a graph.

(c) Are the following assertions true or false?

$F_{0and1} = F_0 \cap F_1$

$F_{0or1} = F_0 \cup F_1$

3. **20 points** Consider all state machines with

$$\textit{Inputs} = \{1, 2, \textit{absent}\} \quad \text{and} \quad \textit{Outputs} = \{1, 2, \textit{absent}\} \quad \text{and} \quad \textit{States} = \{a, b, c, d\}.$$

Assume all these state machines stutter, as usual, when presented with the stuttering input, *absent*.

(a) Give a state machine B that simulates all of these state machines. You will lose points if your machine is more complicated than it needs to be.

(b) Give the simulation relation.

4. **30 points** Consider the functions

$$g\colon Y \to \textit{Reals} \quad \text{and} \quad f\colon \textit{Nats} \to Y.$$

where Y is a set.

(a) Draw a block diagram for $(g \circ f)$, with one block for each of g and f, and label the inputs and output of the blocks with the domain and range of g and f.

(b) Suppose Y is given by

$$Y = [\{1, \cdots, 100\} \to \textit{Reals}]$$

(Thus, the function f takes a natural number and returns a sequence of length 100, while the function g takes a sequence of length 100 and returns a real number.)

Suppose further that g is given by: for all $y \in Y$,

$$g(y) = \sum_{i=1}^{100} y(i) = y(1) + y(2) + \cdots + y(100),$$

and f by: for all $x \in \textit{Nats}$ and $z \in \{1, \cdots, 100\}$,

$$(f(x))(z) = \cos(2\pi z/x).$$

(Thus, x gives the period of a cosine waveform, and f gives 100 samples of that waveform.) Give a one-line Matlab expression that evaluates $(g \circ f)(x)$ for any $x \in \textit{Nats}$. Assume the value of x is already in a Matlab variable called `x`.

(c) Find $(g \circ f)(1)$.

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