EECS20n, Quiz 3 Solution, 3/17/00

The quiz will take 15 minutes. Do your calculations on the sheet.

Please print your name here:

Last Name: ________________ First: ________________ Lab time: ________________

1. Consider each of the functions $x\colon$ *Ints* $\to$ *Comps* given below Is it periodic? If so, what is the period?

   (a) $\forall\, n \in$ *Ints*, $\quad x(n) = e^{i(\omega n + a)}$, where $\omega = 3\pi$ and $a = \pi$.

   Answer
   Yes, it is periodic. The period is the smallest integer $p > 0$ such that $x(n+p) = x(n)$ for all integers n. I.e.,
   $$e^{i(\omega(n+p)+a)} = e^{i(\omega n + a)}.$$
   The left side equals
   $$e^{ia}e^{i\omega n}e^{i\omega p}.$$
   This is equal to the right side if $\omega p = K2\pi$ for some integer K, or if $3\pi p = 2K\pi$ or $p = 2K/3$. The smallest integer p for which this can be true is $p = 2$.

   (b) $\forall\, n \in$ *Ints*, $\quad x(n) = e^{n(i\omega + a)}$, where $\omega = 3\pi$ and $a = \pi$.

   Answer
   No, it is not periodic, because of the factor e^{na}. Following the same method as above, if it were periodic, then its period p would have to be an integer such that
   $$e^{(n+p)(i\omega+a)} = e^{n(i\omega+a)}.$$
   This would require that
   $$p(i\omega + a) = p(i3\pi + \pi) = K2\pi$$
   for some integer K, which is clearly not possible.

2. Use Euler's relation to prove that $\forall\, \theta \in$ *Reals*, $\quad \cos^2(\theta) + \sin^2(\theta) = 1$.

   Answer

   $$\begin{aligned} e^{i\theta}e^{-i\theta} &= (\cos(\theta) + i\sin(\theta))(\cos(\theta) - i\sin(\theta)) \\ &= \cos^2(\theta) + \sin^2(\theta). \end{aligned}$$

   Moreover,
   $$e^{i\theta}e^{-i\theta} = |e^{i\theta}|^2 = 1.$$
   Therefore,
   $$\cos^2(\theta) + \sin^2(\theta)) = 1.$$