EECS20n, Quiz 5 Solution, 4/24/00

1. The fundamental frequency is $\omega_0 = \pi/10$, in units of radians per second. To get this systematically, note that the first cosine has a period of 10 and the second has a period of 20/3. The least common multiple of these is 20, so the fundamental frequency is $2\pi/20 = \pi/10$.

   To get the Fourier series coefficients, just write the signal as a sum of complex exponentials,

   $$x(t) = (1/2)e^{i(\pi/10)2t} + (1/2)e^{-i(\pi/10)2t} + (1/2)e^{i(\pi/10)3t} + (1/2)e^{-i(\pi/10)3t},$$

   from which we can read off the coefficients,

   $$\begin{aligned} X_{-3} &= 1/2 \\ X_{-2} &= 1/2 \\ X_2 &= 1/2 \\ X_3 &= 1/2. \end{aligned}$$

   The rest of the coefficients are zero.

2. The Fourier series coefficients of the output will be the above Fourier series coefficients multiplied by $H(\omega)$ for the corresponding value of ω. At $\omega = 2\pi/10$ and $-2\pi/10$, $H(\omega) = 1$. At $\omega = 3\pi/10$ and $-3\pi/10$, $H(\omega) = -1$. This yields

   $$y(t) = (1/2)e^{i(\pi/10)2t} + (1/2)e^{-i(\pi/10)2t} - (1/2)e^{i(\pi/10)3t} - (1/2)e^{-i(\pi/10)3t},$$

so

$$y(t) = \cos(\pi t/5) - \cos(3\pi t/10).$$