EECS 20. Final Exam
May 20, 2003.

Please use these sheets for your answer. **Write clearly and show your work on the sheets in the back.** Please check that you have 11 numbered pages.

Print your name and lab time below

Name: ______________________________

Lab time: ______________________________

Problem 1 (15):

Problem 2 (20):

Problem 3 (20):

Problem 4 (25):

Problem 5 (20):

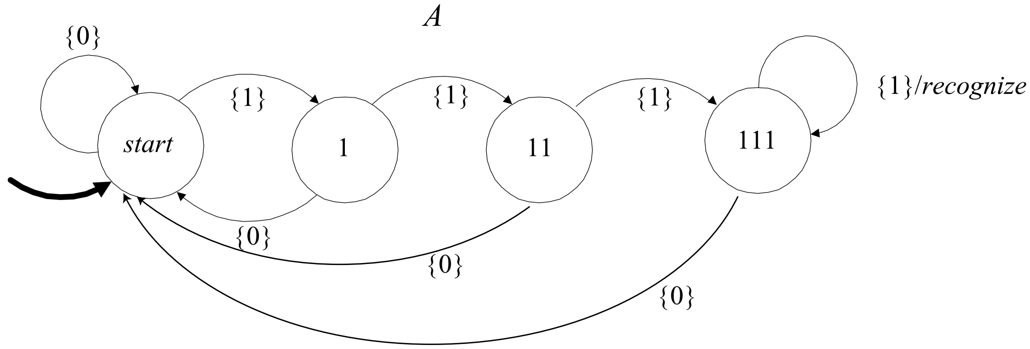
Problem 6 (10):

Problem 7 (10):

Total:

1. **15 points. 5 points for (a), 10 points for (b)**

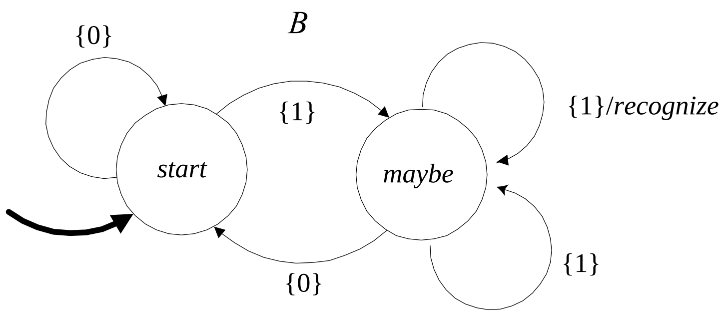
   (a) The deterministic machine A is like the *CodeRecognizer* machine studied in the text and in the homework.



   Let x denote an input signal and y the corresponding output signal. Complete the expression for $y(n)$ below, ignoring stuttering inputs, (i.e. replace the $\cdots$ by an expression involving x)

$$\forall x \in \textit{InputSignals}, \qquad \forall n \in \textit{Naturals}_0,$$
$$y(n) = \begin{cases} \textit{recognize}, & \text{if } \cdots \\ \textit{absent}, & \text{otherwise} \end{cases}$$

   (b) Determine whether the non-deterministic machine B simulates A and write down the relevant simulation relation if it does.



2. **20 points. 5 points for (a), (b), 10 points for (c)** The input signal x and output signal y of an LTI system are related by the differential equation

$$\forall t \in \textit{Reals}, \quad \dot{y}(t) + y(t) = x(t).$$

(a) The frequency response of this system is

$$\forall \omega \in \textit{Reals}, \quad H(\omega) =$$

and the *magnitude* and *phase* response for $\omega = 0, \pm 1$, and $\omega \to \pm\infty$ are:

(b) The impulse response of this system is

$$\forall t \in \textit{Reals} \quad h(t) = \begin{cases} \quad , & \text{if } t < 0 \\ \quad , & \text{if } t > 0 \end{cases}$$

Hint: The Fourier transform of the signal $x(t) = 0, t < 0; x(t) = e^{-t}, t \geq 0$ is $\forall \omega, X(\omega) = [1 + i\omega]^{-1}$.

(c) Now consider an LTI system whose impulse response $g = h * h$, where h is as in (2b). Let G be the frequency response of this system. Then

$$\forall \omega \in \textit{Reals}, \quad G(\omega) = \qquad ,$$

$$|G(1)| = \qquad , \quad \angle G(1) =$$

$$\forall t \in \textit{Reals}, \quad g(t) = \begin{cases} \quad , & \text{if } t < 0 \\ \quad , & \text{if } t > 0 \end{cases}$$

3. **20 points, 4 points each part** Let M be a deterministic state machine with input and output alphabet $\{0, 1, absent\}$. State whether the following propositions are true or false.

(a) Suppose M has a *finite* number of states. Let $y = (y(0), y(1), \cdots,)$ be the output signal corresponding to the input signal $x = (0, 0, 0, \cdots)$ (all zero sequence). Then the output signal y must be eventually periodic, i.e. there are integers N, p such that $\forall n > N, y(n + p) = y(n)$.

Answer:

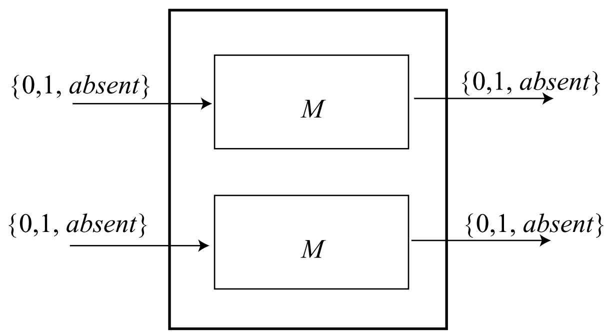
(b) Suppose the output signal y of M is related to its input signal x by: $\forall n \geq 0$,

$$y(n) = \begin{cases} 1, & \text{if } x(0), \cdots, x(n) \text{ contain an } unequal \text{ number of 0s and 1s,} \\ 0, & \text{otherwise} \end{cases}$$

Then M has an *infinite* number of states.

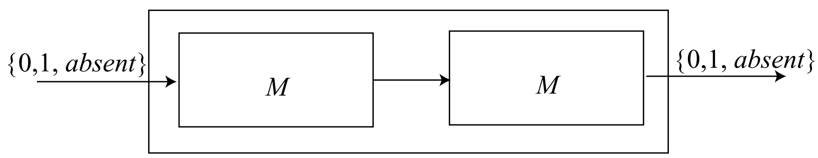
Answer:

(c) Suppose all the states of M are *reachable* (from the initial state). Then all states of the side-by-side composition of M with itself are reachable. The composition is shown below.



Answer:

(d) Suppose all the states of M are *reachable* (from the initial state). Then all states of the cascade composition of M with itself are reachable. The composition is shown below.



Answer:

(e) Suppose N is another deterministic state machine that simulates M. Then M simulates N.

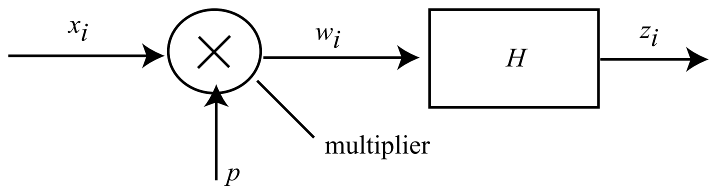
Answer:

4. **25 points, 5 points each part** In the block diagram below of a sampling and reconstruction system, the input signal x_i : *Reals* $\to$ *Complex* is multiplied by the periodic impulse train p to produce the sampled signal w_i. Here

$$\forall t \in \textit{Reals}, \quad p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT),$$

The ideal reconstruction filter has frequency response H:

$$\forall \omega, \quad H(\omega) = \begin{cases} T, & \text{if } |\omega| < \pi/T \\ 0, & \text{otherwise} \end{cases}$$



Assume below that the sampling frequency is $f = 8,000$ Hz, $T = 125\mu$s.

Note: Answers to questions below do not require much calculation

(a) The Fourier transform of p is

$$\forall \omega \in \textit{Reals}, \quad P(\omega) = \qquad\qquad\qquad\qquad .$$

(b) In terms of X_i, the Fourier Transform of x_i, the Fourier Transform of w_i is

$$\forall \omega \in \textit{Reals}, \quad W_i(\omega) = \qquad\qquad\qquad\qquad .$$

and the Fourier Transform of z_i is

$$\forall \omega \in \textit{Reals}, \quad Z_i(\omega) = \qquad\qquad\qquad\qquad .$$

(c) Suppose $\forall t$, $x_1(t) = \cos(2\pi \times 1000t)$. Then $\forall \omega \in$ *Reals*,

$$\begin{aligned} X_1(\omega) &= \\ W_1(\omega) &= \\ Z_1(\omega) &= \end{aligned}$$

(d) Suppose $\forall t$, $x_2(t) = \cos(2\pi \times 7000t)$. Then $\forall \omega \in$ *Reals*,

$$\begin{aligned} X_2(\omega) &= \\ W_2(\omega) &= \\ Z_2(\omega) &= \end{aligned}$$

(e) Suppose $\forall t$, $x_3(t) = \cos(2\pi \times 1000t) - \cos(2\pi \times 7000t)$. Then $\forall \omega \in$ *Reals*,

$$Z_3(\omega) =$$

and $\forall t \in$ *Reals*,

$$z_3(t) =$$

5. **20 points, 5 points each part** The step input for a continuous time system is defined as $x(t) = 0, t < 0, x(t) = 1, t \geq 0$; and for a discrete time system it is defined as $x(n) = 0, n < 0, x(n) = 1, n \geq 0$.

   (a) If the impulse response of a continuous time LTI system is

$$\forall t, \quad h(t) = \begin{cases} 0, & t < 0 \\ e^{-t}, & t \geq 0 \end{cases}$$

   its step response is

$$s(t) = \begin{cases} \qquad\qquad\qquad\qquad , & t < 0 \\ \qquad\qquad\qquad\qquad , & t \geq 0 \end{cases}$$

   (b) If the impulse response of a continuous time LTI system is

$$h(t) = \begin{cases} e^{-|t|}, & t < 0 \\ 0, & t \geq 0 \end{cases}$$

   its step response is

$$s(t) = \begin{cases} \qquad\qquad\qquad\qquad , & t < 0 \\ \qquad\qquad\qquad\qquad , & t \geq 0 \end{cases}$$

   (c) If the impulse response of a discrete time LTI system is

$$h(n) = \begin{cases} 1, & n = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

   its step response is

$$s(n) = \begin{cases} \\ \\ \\ \end{cases}$$

   (d) If the impulse response of a discrete time LTI system is

$$h(n) = \begin{cases} 1, & n = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

   its step response is

$$s(n) = \begin{cases} \\ \\ \\ \end{cases}$$

6. **10 points, 3 points for (a)-(c), 1 point for (d)** For each continuous time signals x_i, write down its Fourier transform X_i

(a) $\forall t, x_1(t) = e^{i20t}$.

$\forall \omega, X_1(\omega) =$

(b) $\forall t, x_2(t) = 1, |t| < T; x_2(t) = 0, |t| > T$.

$\forall \omega, X_2(\omega) =$.

(c) $\forall t, x_3(t) = x_1(t) \times x_2(t),$ where x_1, x_2 are as above.

$\forall \omega, X_3(\omega) =$.

(d) The unit of ω above is

7. **10 points** For each of the following discrete-time systems with input signal x and output signal y, state whether it is linear (L), time-invariant (T), linear and time-invariant (LTI), or none (N).

$\forall n, \quad y(n) = x(2-n)$ **Answer:**

$\forall n, \quad y(n) = [x(n-1)]^2$ **Answer:**

$\forall n, \quad y(n) = \sum_{m=-\infty}^{\infty} 0.5^{|m|} x(n-m)$ **Answer:**

$\forall n, \quad y(n) = x(2-n) + x(n-2)$ **Answer:**

$\forall n, \quad y(n) = n^2 x(n)$ **Answer:**

This page is for work

This page is for work