EECS 20. Midterm No. 2
April 11, 2003.

Please use these sheets for your answer and your work. Use the backs if necessary. **Calculators are NOT allowed. Write clearly and put a box around your answer, and show your work.**

Print your name and lab day and time below

Name: ______________________________

Lab time: ______________________________

Problem 1:

Problem 2:

Problem 3:

Problem 4:

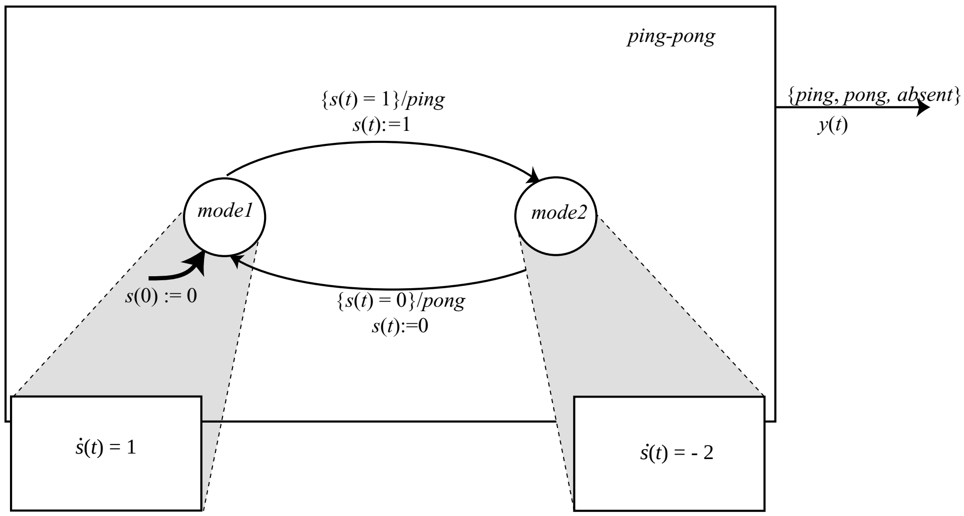
Problem 5:

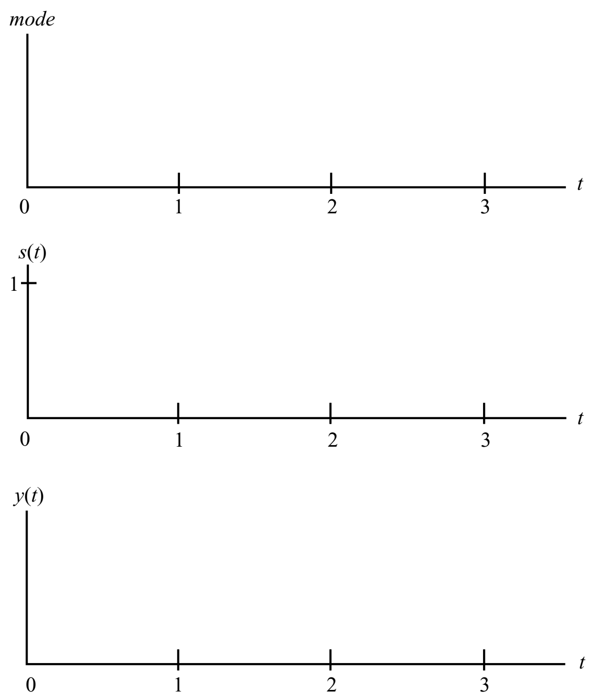
Total:

1. **15 points** For the following hybrid system sketch in the graphs below

   (a) (**10 points**) the state trajectory (both the mode and the continuous state) and

   (b) (**5 points**) the output signal y for $0 \leq t \leq 3$.





2. **15 points, 5 points each part**
Give the units of period and frequency below

(a) Consider the discrete-time signal x given by

$$\forall n \in Integers, \quad x(n) = \cos(\omega n).$$

For what values of ω is x periodic, and what is the period?

(b) Consider the discrete-time signal x given by

$$\forall n \in Integers, \quad x(n) = 1 + \cos(4\pi n/9).$$

What is its period p and what is its fundamental frequency?

This signal has the Fourier series representation

$$\forall n, \quad y(n) = A_0 + \sum_{k=1}^{\lfloor p/2 \rfloor} A_k \cos(k\omega_0 n + \phi_k).$$

Identify $\omega_0, A_0, A_k, \phi_k$.

(c) Consider the continuous-time periodic signal y given by

$$\forall t \in Reals, \quad y(t) = \cos(5t) + \sin(3t).$$

What is its period and what is its fundamental frequency?

The Fourier series representation of y above is

$$\forall t, \quad y(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k).$$

Identify ω_0, A_0, A_k, ϕ_k.

3. **15 points, 3 points each part** Consider the following discrete-time systems with input $x : Integers \to Reals$ and output signal $y : Integers \to Reals$. For each system, state whether it is linear (L), time-invariant (T), both (LTI), or neither (N). No proof or counterexample is required.

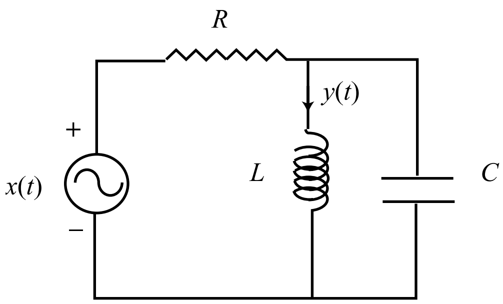
(a) $\forall n, \quad y(n) = x(-n).$

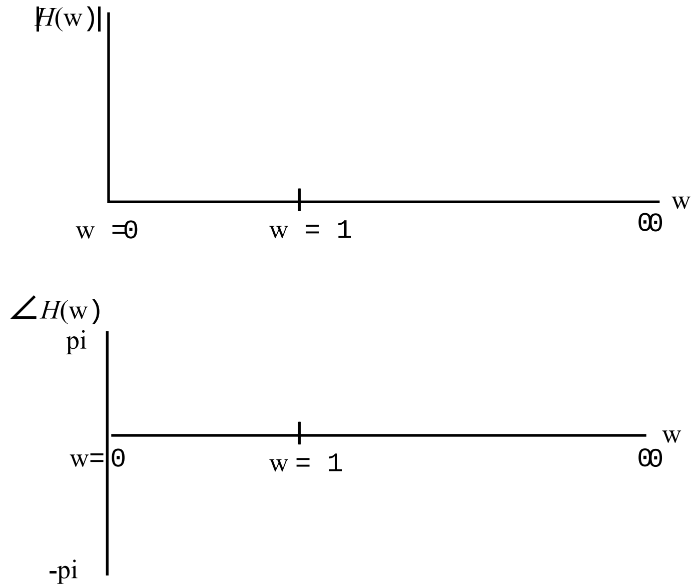
(b) $\forall n, \quad y(n) = [x(n) + x(n-1)]^2.$

(c) $\forall n, \quad y(n) = n[x(n) + x(n-1)].$

(d) $\forall n, \quad y(n) = x(2n).$

(e) $\forall n, \quad y(n) = [x(n) + x(n+1)]/2.$





4. **20 points** The R, L, C circuit in the figure has for its input signal the voltage x and its output signal is the inductor current y. From Kirchhoff's law one can determine that these signals are related by the differential equation

$$\forall t, \quad RLC\frac{d^2y(t)}{dt^2} + L\frac{dy(t)}{dt} + Ry(t) = x(t).$$

(a) **6 points** Find the frequency response $H : Reals \to Complex$ of this system.

(b) **7 points** Obtain an expression for the amplitude response and the phase response, assuming $R = L = C = 1$.

(c) **7 points** Sketch the amplitude response and the phase response in the graphs above. Carefully mark the values for $\omega = 0, 1$ and $\omega \to \infty$.

5. **15 points, 5 points each part**
Fill in the blanks:

(a) The five roots of $z^5 = 1$ are:

(b) $\forall t, \quad \cos(\omega t) + \cos(\omega t + \pi/2) = Re\{Ae^{i[\omega t + \phi]}\}$

in which $A =$ and $\phi =$. (A should be a positive real number)

(c) The polar representation of the following numbers are:

$1 + i =$ $1 - i =$ $[1 + i]^{-1} =$

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