EECS20n, Quiz 3 Solution, 3/5/03

A single-input single-output system has the $[A, b, c^T, d]$ representation given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad c^T = [1\ 0\ 0], \quad d = 0.$$

1. Calculate $A^n, n \geq 0$, by carrying out the matrix multiplications.

$$A^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A^n = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, n \geq 3.$$

2. Recall that the impulse response is given by $h : Naturals_0 \to Reals$, in which $h(0) = d, h(n) = c^T A^{n-1} b, n \geq 1$. Find the impulse response for the system given above.

   By substituting for A^n we get:

$$h(0) = 0, h(1) = 1, h(n) = 0, n \geq 2.$$

3. For the input $x : Naturals_0 \to Reals$ given by $x(1) = 1$ and $x(n) = 0, n \neq 1$, find the zero-state response $y : Naturals_0 \to Reals$.

   We have

$$\begin{aligned} \forall n \geq 0, y(n) &= \sum_{k=0}^{n} h(n-k)x(k) \\ &= h(n-1) = \begin{cases} 1, & n = 2 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$