# EECS20n, Quiz 4 Solution, 3/19/03

1. The signal $s : Reals \to Reals$ is given by

$$\forall t \in Reals, \quad s(t) = 2 + \sin 2\pi t + \sin 3\pi t.$$

   (a) What is the period of $s$ in seconds (assume $t$ is in seconds)?
   The period is 2 s. We can see this by rewriting $s$ as

$$\forall t, \quad s(t) = \sin 2\pi \times 1 \times t + \sin 2\pi \times 3/2 \times t,$$

   so the fundamental frequency is $f_0 = gcd(1, 3/2) = 1/2$ and the period is $1/f_0 = 2$ s.

   (b) Write down the Fourier series expansion of $s$ in the form

$$\forall t, \quad s(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(2\pi k f_0 t + \phi_k),$$

   i.e. identify $f_0$ and the coefficients, $A_0, A_k, \phi_k$.
   Rewrite the signal as

$$\forall t, \quad s(t) = 2 + \cos(2\pi 2 f_0 t - \frac{\pi}{2}) + \cos(2\pi 3 f_0 t - \frac{\pi}{2}).$$

   Comparing coefficients with Fourier series representation gives:

$$f_0 = \frac{1}{2}; A_0 = 2, A_2 = 1, A_3 = 1, A_k = 0, \text{otherwise}; \phi(k) = -\frac{\pi}{2}, \text{ all } k.$$

   (c) In the following $x$ is a discrete-time signal $x : Integers \to Reals$. For each case determine whether $x$ is periodic and if it is periodic find its period (in samples).

      i.

$$\forall n, \quad x(n) = 1 + \cos(2\pi \times 5n).$$

      The period $p$ is the smallest integer $p$ such that $2\pi 5p$ is a multiple of $2\pi$, which gives $p = 1$ sample. Indeed, $\forall n, \quad \cos(2\pi \times 5n) = 1$. The signal is periodic.

      ii.

$$\forall n, \quad x(n) = \sin(2\pi \times \frac{5}{7} n).$$

      The period is the smallest integer $p$ such that $2\pi \frac{5}{7} p$ is a multiple of $2\pi$, which gives $p = 7$ samples. The signal is periodic.