EECS20n, Quiz 2, 03/08/04, Solution

1. **5 points** For each of the following definitions of a function $f : R^3 \rightarrow R$, determine whether it is linear (L) or non-linear (N).

   (a) $\forall x, \quad f(x) = 0$ — $\boxed{\text{L}}$

   (b) $\forall x, \quad f(x) = 1$ — $\boxed{\text{NL}}$

   (c) $\forall x = (x_1, x_2, x_3), \quad f(x) = x_1 + 2x_2$ — $\boxed{\text{L}}$

   (d) $\forall x = (x_1, x_2, x_3), \quad f(x) = x_1 + x_2 + 1$ — $\boxed{\text{NL}}$

   (e) $\forall x = (x_1, x_2, x_3), \quad f(x) = x_1^2$ — $\boxed{\text{NL}}$

2. Consider the three-dimensional SISO system whose $[A, b, c, d]$ representation is

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad c^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \quad d = 1$$

   (a) **5 points** Calculate $A^n, n \geq 0$.

$$A^0 = I, \quad A^1 \text{ is given }, \quad A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad A^n = 0, n \geq 3.$$

   (b) **5 points** Find the zero-state impulse response $h(n), n \geq 0$.
   Substitution in $h(0) = d, h(n) = c^T A^{n-1} b, n \geq 1$ gives
   $\boxed{h(0) = 1,\ h(1) = 1,\ h(2) = 2,\ h(3) = 3,\ h(n) = 0, n \geq 4}$