EECS20n, Quiz 7 Solution, 04/23/04

Recall the definitions of DTFT and CTFT:

$$
\begin{aligned}
x \in \textit{DiscSignals} \quad &\rightarrow \quad \forall \omega \in R, X(\omega) = \sum_{-\infty}^{\infty} x(k) e^{-i\omega k} \\
X \in \textit{ContPeriodic}_{2\pi} \quad &\rightarrow \quad x(k) = \frac{1}{2\pi} \int_0^{2\pi} X(\omega) e^{i\omega k} \\
x \in \textit{ContSignals} \quad &\rightarrow \quad \forall \omega \in R, X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \\
X \in \textit{ContSignals} \quad &\rightarrow \quad \forall t \in R, x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega
\end{aligned}
$$

1. If $x \in \textit{ContSignals}$ is given by $\forall t \in R, x(t) = \delta(t-1)$, its CTFT is

$$\forall \omega, X(\omega) = \int_{-\infty}^{infty} \delta(t-1) e^{-i\omega t} dt = \boxed{e^{-i\omega}}$$

2. If $X \in \textit{ContSigals}$ is given by $\forall \omega \in R, X(\omega) = \delta(\omega - 20) + \delta(\omega + 20)$, its InverseCTFT is

$$
\begin{aligned}
\forall t, x(t) \quad &= \quad \frac{1}{2\pi} \int_{-\infty}^{\infty} [\delta(\omega - 20) + \delta(\omega + 20)] e^{i\omega t} d\omega \\
&= \quad \frac{1}{2\pi} [e^{i20t} + e^{-i20t}] = \boxed{\tfrac{1}{\pi} \cos(20t)}
\end{aligned}
$$

3. If $x \in \textit{DiscSignals}$ is given by $\forall k \in \textit{Ints}, x(k) = (0.5)^k, k \geq 0; x(k) = 0, k < 0$, its DTFT is

$$\forall \omega, X(\omega) = \sum_{0}^{\infty} (0.5)^k e^{-i\omega k} = \boxed{\tfrac{1}{1 - 0.5 e^{-i\omega}}}$$

4. If $X \in \textit{PeriodicSignals}_{2\pi}$ is given by $\forall \omega \in R, X(\omega) = 1$, its InverseDTFT is

$$\forall k, x(k) = \frac{1}{2\pi} \int_0^{2\pi} 1 \times e^{i\omega k} d\omega = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

because for $k \neq 0$,

$$\int_0^{2\pi} e^{i\omega k} d\omega = 0.$$