EECS20n, Quiz 8 Solution

Let the continuous-time signal c given by

$$\forall\, t \in \textit{Reals}, \quad c(t) = 2\cos(\omega_c t)$$

be a carrier wave for a radio signal. Let x given by

$$\forall\, t \in \textit{Reals}, \quad x(t) = 2\cos(\omega_x t)$$

be the signal to be carried by that radio signal (that it, it is a highly simplified stand-in for, say, a voice signal). To be concrete, let $\omega_c = 2\pi \cdot 8000$ radians/second, and $\omega_x = 2\pi \cdot 400$ radians/second.

1. Find and sketch the CTFT Y of y where

   $$\forall\, t \in \textit{Reals}, \quad y(t) = c(t)x(t).$$

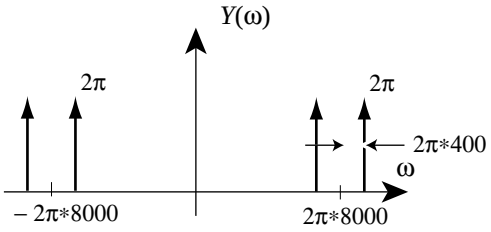
   Label your sketch carefully. **Hint**: The CTFT of $e^{i\omega_0 t}$ is $2\pi\delta(\omega - \omega_0)$.

   Answer Using $2\cos(\omega t) = e^{i\omega t} + e^{-i\omega t}$,

   $$\begin{aligned} y(t) &= [e^{i\omega_c t} + e^{-i\omega_c t}][e^{i\omega_x t} + e^{-i\omega_x t}] \\ &= e^{i(\omega_c+\omega_x)t} + e^{i(\omega_c-\omega_x)t} + e^{i(-\omega_c+\omega_x)t} + e^{i(-\omega_c-\omega_x)t}. \end{aligned}$$

   Using the Hint and the fact that the CTFT is linear, we have

   $$\begin{aligned} \forall\omega, Y(\omega) &= 2\pi[\delta(\omega - (\omega_c + \omega_x)) + \delta(\omega - (\omega_c - \omega_x)) \\ &+ \delta(\omega - (-\omega_c + \omega_x)) + \delta(\omega - (-\omega_c - \omega_x))] \end{aligned}$$

2. Let y from part 1 be the input to an LTI system with frequency response H where

   $$\forall\, \omega \in \textit{Reals}, \quad H(\omega) = \begin{cases} 0 & \text{if } \omega \le 0 \\ 1 & \text{if } \omega > 0 \end{cases}$$

   Find the output u as a function of t.

   Answer Since

   $$\begin{aligned} \forall\omega, U(\omega) &= H(\omega)Y(\omega) \\ &= 2\pi[\delta(\omega - (\omega_c + \omega_x)) + \delta(\omega - (\omega_c - \omega_x))], \end{aligned}$$

   $$\forall t, u(t) = e^{i(\omega_c+\omega+x)t} + e^{i(\omega_c-\omega_x)t}.$$