EECS 20n, Diagnostic Takehome Exam, 1/21/04, Due 1/26/04 in Class

This 'takehome' exam is to provide feedback to you and to me about how well prepared you are with the prerequiste math for this course. The exam will take about one hour. **Write clearly, put a box around your answer, and show your work.**

Please print your name and lab time here: Last Name ———— First ———— Lab time ————

1. Let $z_1 = 3 + 4i$ and $z_2 = 5 + 12i$ be two complex numbers. Then

   (a) $z_1 + z_2 =$

   (b) $z_1 * z_2 =$

   (c) $z_2/z_1 =$

2. (a) $e^{i\pi} =$

   (b) Show why $\cos 3\theta = 4\cos^3 \theta - 3\cos\theta$.

   (c) Express $\sin 3\theta$ in terms of $\sin\theta$.

   (a) Does $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converge? Why?

   (b) What is
   $\lim_{x\to 0} \frac{\sin x}{x} =$

3. Solve the following first order linear differential equation:

$$\frac{dy}{dx} = 2x + 1,$$

with the initial condition $y(0) = 0$. What is $y(1)$? Plot $y(x)$ for $0 \le x \le 1$.

4. Let A be the matrix
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

(a) Verify $A^2 = A$.

(b) Is it invertible? What is A^{-1}?

(c) Find all its eigenvalues.