EECS 20n: Structure and Interpretation of Signals and Systems QUIZ 4
Department of Electrical Engineering and Computer Sciences 20 April 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________ FIRST Name ______________ Lab Time ______________

- This quiz should take you up to 15 minutes to complete.
- Please limit your work to the space provided for each problem.
  No other work will be considered in grading your quiz.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- Full credit will be given *only* to work that is clearly explained.
- The following may be of potential use to you:
  - Complex exponential Fourier series expressions for a periodic discrete-time signal having period p:

$$\begin{aligned} x(n) &= \sum_{k=\langle p\rangle} X_k\, e^{ik\omega_0 n} \\ X_k &= \frac{1}{p}\sum_{n=\langle p\rangle} x(n)\, e^{-ik\omega_0 n}\ , \end{aligned}$$

    where $p = \frac{2\pi}{\omega_0}$ and $\langle p\rangle$ denotes a suitable contiguous discrete interval of length p (for example, $\sum_{k=\langle p\rangle}$ can denote $\sum_{k=0}^{p-1}$).

  - Complex exponential Fourier series expressions for a periodic continuous-time signal having period p:

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} X_k\, e^{ik\omega_0 t} \\ X_k &= \frac{1}{p}\int_{\langle p\rangle} x(t)\, e^{-ik\omega_0 t} dt\ , \end{aligned}$$

    where $p = \frac{2\pi}{\omega_0}$ and $\langle p\rangle$ denotes a suitable continuous interval of length p (for example, $\int_{\langle p\rangle}$ can denote $\int_0^p$).

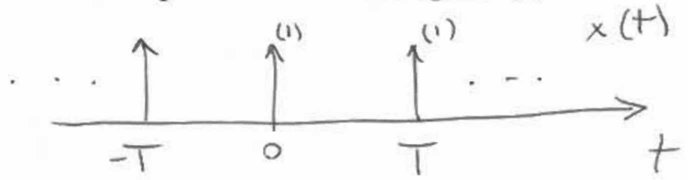
You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

Problem 1 (10 points) Consider a continuous-time periodic signal $x(t)$ characterized by a train of impulses, as follows:

$$x(t) = \sum_{l=-\infty}^{+\infty} \delta(t - lT),$$

where T is a positive, real constant denoting the spacing between any pair of adjacent impulses.

Identifyi~~ng~~ the period and fundamental frequency of $x(t)$, and then determine, for all integer values of k, the coefficients X_k in the complex exponential Fourier series representation of the signal $x(t)$.



$P = T$

$\omega_0 = \frac{2\pi}{T}$

$$X_k = \frac{1}{P} \int_{\langle P \rangle} x(t) e^{-ik\omega_0 t} dt$$

Select $\left[-\frac{T}{2}, \frac{T}{2}\right)$ as interval of integration. In this interval, $x(t) = \delta(t)$ $\Longrightarrow$

$$X_k = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) e^{-ik\omega_0 t} dt = \frac{1}{T} \Longrightarrow$$

$$\underline{X_k = \frac{1}{T}} \quad \forall k \text{ integer}$$

Problem 2 (10 points) Determine all the coefficients X_k in the complex exponential Fourier series representation of the periodic discrete-time signal shown below:

$$x(n) = \cos\left(\frac{2\pi n}{3}\right) + \sin(\pi n) \ .$$

Hint: What is the period p of $x(n)$? What is the fundamental frequency ω_0 of $x(n)$?

$\sin(\pi n)$ always equals 0 and is ignored.

$\cos(\frac{2\pi n}{3})$ has period 3 $\Rightarrow$

$x(n)$ has period 3. $\Rightarrow$

$p = 3, \quad \omega_0 = \frac{2\pi}{p} = \frac{2\pi}{3}$

X_k's can be written w/o much algebra.

$$x(n) = \underbrace{\frac{1}{2} e^{i\frac{2\pi}{3}n} + \frac{1}{2} e^{-i\frac{2\pi}{3}n}}_{\cos(\frac{2\pi}{3}n)} + 0$$

$\frac{1}{2} e^{i\frac{2\pi}{3}n}$ corresponds to $k = 1 \Rightarrow X_1 = \frac{1}{2}$

$\frac{1}{2} e^{-i\frac{2\pi}{3}n}$ corresponds to $k = -1 \Rightarrow X_{-1} = \frac{1}{2}$.

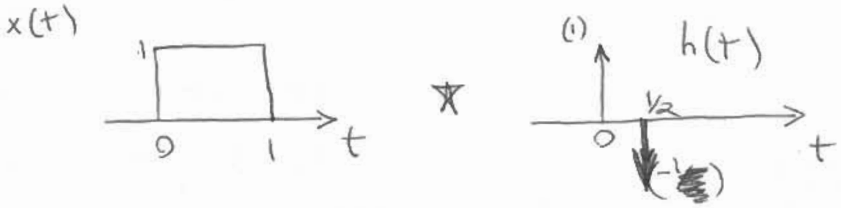
Problem 3 (10 points) Consider a continuous-time LTI system having the following impulse response:

$$h(t) = \delta(t) - \delta(t - \frac{1}{2}) .$$

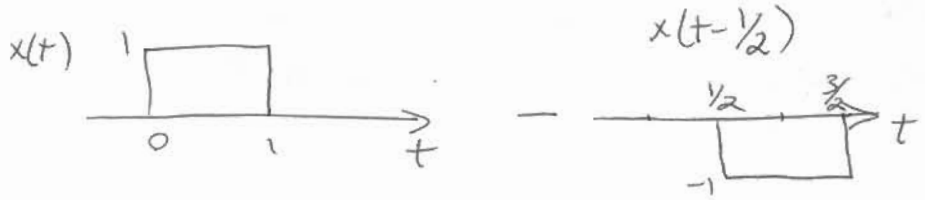
The following input signal is applied to the LTI system:

$$x(t) = \begin{cases} 1 & 0 < t \leq 1 \\ 0 & \text{elsewhere} \end{cases} .$$

Determine and plot the output signal $y(t)$ corresponding to the input signal $x(t)$, and fully and clearly label all the salient features of the plot.



$$y(t) = x(t) - x(t - \frac{1}{2})$$



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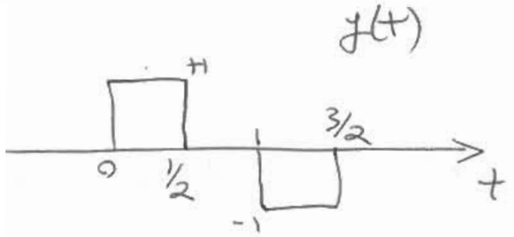
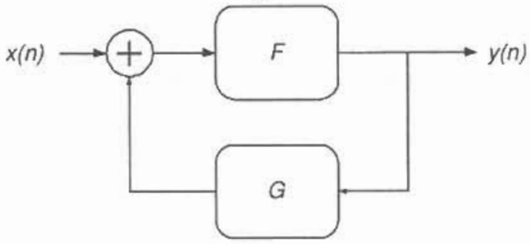


Problem 4 (10 points) Consider the feedback interconnection of discrete-time LTI systems F and G shown in the diagram below:



System F is characterized by its frequency response $F(\omega)$, given by $F(\omega) = \dfrac{1}{2\cos(\omega)}$. System G is characterized by its impulse response $g(n)$, given by $g(n) = \delta(n-1)$.

(a) (7 points) Find the impulse response $h(n)$ of the composite feedback system.

$H(\omega) = \dfrac{F(\omega)}{1-F(\omega)G(\omega)}$, where $H(\omega)$ is the freq resp. of the feedback interconnection.

$g(n) = \delta(n-1) \Rightarrow G(\omega) = e^{-i\omega} \Rightarrow H(\omega) = \dfrac{1/2\cos(\omega)}{1-\dfrac{e^{-i\omega}}{2\cos(\omega)}} = \dfrac{1}{2\cos(\omega) - e^{-i\omega}}$

Recall $\cos(\omega) = \dfrac{e^{i\omega}+e^{-i\omega}}{2} \Rightarrow 2\cos(\omega) = e^{i\omega}+e^{-i\omega}$

$H(\omega) = \dfrac{1}{e^{i\omega}+e^{-i\omega}-e^{-i\omega}} = e^{-i\omega} \Rightarrow H(\omega) = e^{-i\omega}$ $\quad h(n) = \delta(n-1)$

(b) (3 points) Find the output signal $y(n)$ corresponding to the following input signal:

$$x(n) = e^{i\frac{2\pi}{5}n}u(n) .$$

$\left.\begin{matrix} y(n) = (x * h)(n) \\ h(n) = \delta(n-1) \end{matrix}\right\} \Rightarrow y(n) = x(n-1)$ simple delayed version.

Note: a truncated complex exponential is not, in general, an eigenfctn of an LTI system.