EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 3 SOL.
Department of Electrical Engineering and Computer Sciences 13 April 2006
UNIVERSITY OF CALIFORNIA BERKELEY

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Lab Time 365/24/7

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2	20	20
Total	**45**	45

Basic Formulas:

Discrete Fourier Series (DFS) Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n} \ ,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable discrete interval of length p (i.e., an interval containing p contiguous integers). For example, $\displaystyle\sum_{k=\langle p \rangle}$ may denote $\displaystyle\sum_{k=0}^{p-1}$ or $\displaystyle\sum_{k=1}^{p}$.

Continuous-Time Fourier Series (FS) Complex exponential Fourier series synthesis and analysis equations for a periodic continuous-time signal having period p:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{ik\omega_0 t} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) \, e^{-ik\omega_0 t} dt \ ,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable continuous interval of length p. For example, $\displaystyle\int_{\langle p \rangle}$ can denote $\displaystyle\int_0^p$.

Frequency Response of a DT LTI System Consider a real, discrete-time LTI system having impulse response $h : \mathbb{Z} \to \mathbb{R}$. The frequency response $H : \mathbb{R} \to \mathbb{C}$ of the system is given by:

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n) \, e^{-i\omega n} \, , \forall \omega \, .$$

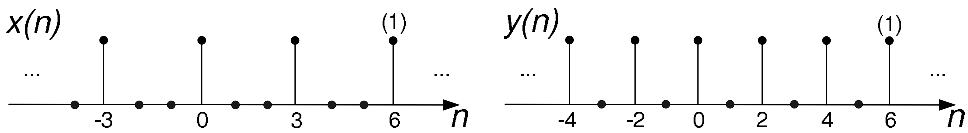
Q3.1 (20 Points) (a) Consider a real, discrete-time system

$$\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$$

having input signal x and corresponding output signal y, as described below:

$$\begin{cases} x(n) = \sum_{l=-\infty}^{\infty} \delta(n - 3l)\,, & \forall n \\ y(n) = \sum_{l=-\infty}^{\infty} \delta(n - 2l)\,, & \forall n\,. \end{cases}$$

(i) Could the system F be LTI? If your answer is "Yes," then determine as much as you can about the frequency response F of the system. Explain your reasoning succinctly, but clearly and convincingly.



The system F cannot be LTI. The input x is periodic with period $p_x = 3$ (and fundamental frequency $\omega_{0x} = 2\pi/3$), whereas the output y is periodic with period $p_y = 2$ (and fundamental frequency $\omega_{0y} = 2\pi/2 = \pi$). An LTI system cannot produce an output y containing a frequency component which is absent from the input x.

(ii) Could the system F be memoryless? Explain your reasoning succinctly, but clearly and convincingly.

The system F cannot be memoryless. Note, for example, that $x(1) = x(2) = 0$, whereas $0 = y(1) \neq y(2) = 1$.

(b) Consider a real, discrete-time system

$$\mathsf{G} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$$

having input signal x and corresponding output signal y, as described below:

$$\begin{cases} x(n) = 1 + \cos\left(\frac{\pi}{3}n\right), \forall n \\ y(n) = 1 + \cos\left(\frac{\pi}{4}n\right), \forall n\,. \end{cases}$$

(i) Could the system be LTI? If your answer is "Yes," then determine as much as you can about the frequency response G of the system. Explain your reasoning succinctly, but clearly and convincingly.

The system G cannot be LTI. The output signal y contains frequency components at $\pm\frac{\pi}{4}$, which are absent from the input signal x.

(ii) Could the system be memoryless? Explain your reasoning succinctly, but clearly and convincingly.

The system G cannot be memoryless, because $x(0) = x(6) = 2$, whereas $2 = y(0) \neq y(6) = 1$.

(c) Consider a real, discrete-time system

$$\mathsf{H} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$$

having input signal x and corresponding output signal y, as described below:

$$\begin{cases} x(n) = 1 + \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{2\pi}{3}n\right), \forall n \\ y(n) = \frac{1}{2}\cos\left(\frac{2\pi}{3}n\right), \forall n\,. \end{cases}$$

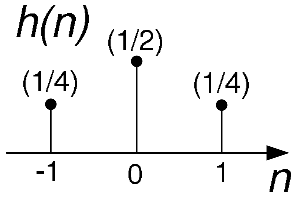
Could the system be LTI? If your answer is "Yes," then determine as much as you can about the frequency response H of the system. Explain your reasoning succinctly, but clearly and convincingly.

The system H could be LTI. The elimination of the input components at DC (i.e., frequency 0) and $\pm\frac{\pi}{3}$ could be implemented by frequency-selective LTI filtering. The input-output behavior is not inconsistent with any property of LTI systems.

If H is indeed LTI, then we can conclude that the frequency response H has the following values: $H(0) = H\left(\pm\frac{\pi}{3}\right) = 0$, and $H\left(\pm\frac{2\pi}{3}\right) = \frac{1}{2}$.

Note that whatever the impulse response is, the frequency response H must be real-valued at frequencies 0, $\pm\frac{\pi}{3}$, and $\pm\frac{2\pi}{3}$, and the phase response $\angle H$ must be zero at these frequencies.

Q3.2 (20 Points) Consider a discrete-time LTI system whose impulse response h is shown below.



The impulse response sample values are zero for all $|n| > 1$. If the input x to the LTI system is

$$x(n) = \frac{1 + (-1)^n}{2} + \cos\left(\frac{2\pi}{3} n\right) + \sin\left(\frac{\pi}{2} n + \frac{\pi}{5}\right) , \quad \forall n ,$$

determine an expression for $y(n)$, the sample values of the corresponding output signal y. Simplify your final expression. Explain your work succinctly, but clearly and convincingly.

First find the frequency response H, which is

$$\begin{aligned} H(\omega) &= \sum_{n=-\infty}^{\infty} h(n)\, e^{-i\omega n} = h(-1)\, e^{i\omega} + h(0) + h(1)\, e^{-i\omega} \\ &= \frac{1}{2} + \frac{1}{4}\left(e^{i\omega} + e^{-i\omega}\right) = \frac{1}{2}(1 + \cos\omega) . \end{aligned}$$

Note that $H(\omega) \in \mathbb{R}_+$, $\forall\omega$. Therefore, $\angle H(\omega) = 0$, $\forall\omega$. Also, note that $H(\omega) = H(-\omega)$, $\forall\omega$. The input signal contains a DC component, i.e., $\frac{1}{2}$; a component at frequency π, i.e., $\frac{1}{2}(-1)^n = \frac{1}{2} e^{i\pi n}$; a component at frequencies $\pm\frac{2\pi}{3}$, i.e., $\cos\left(\frac{2\pi}{3} n\right)$; and a component at frequencies $\pm\frac{\pi}{2}$, i.e., $\sin\left(\frac{\pi}{2} n + \frac{\pi}{5}\right)$. Evaluating the frequency response at these frequencies, we note that:

$$H(0) = 1, \quad H(\pi) = 0, \quad H\left(\pm\frac{2\pi}{3}\right) = \frac{1}{4}, \quad \text{and } H\left(\pm\frac{\pi}{2}\right) = \frac{1}{2}.$$

Therefore, the output is

$$y(n) = \frac{1}{2} + \frac{1}{4}\cos\left(\frac{2\pi}{3} n\right) + \frac{1}{2}\sin\left(\frac{\pi}{2} n + \frac{\pi}{5}\right) .$$

Note that there is no phase influence in the $\sin$ term, because the system's phase response is identically zero at all frequencies.