

LAST Name \_\_\_\_\_ FIRST Name \_\_\_\_\_

Lab Time \_\_\_\_\_

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct.
- Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	
1	20	
2	20	
<b>Total</b>	<b>45</b>	

### Basic Formulas:

**Discrete Fourier Series (DFS)** Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period  $p$ :

$$x(n) = \sum_{k=\langle p \rangle} X_k e^{ik\omega_0 n} \quad \longleftrightarrow \quad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) e^{-ik\omega_0 n} ,$$

where  $p = \frac{2\pi}{\omega_0}$  and  $\langle p \rangle$  denotes a suitable discrete interval of length  $p$  (i.e., an

interval containing  $p$  contiguous integers). For example,  $\sum_{k=\langle p \rangle}$  may denote  $\sum_{k=0}^{p-1}$

or  $\sum_{k=1}^p$ .

**Continuous-Time Fourier Series (FS)** Complex exponential Fourier series synthesis and analysis equations for a periodic continuous-time signal having period  $p$ :

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t} \quad \longleftrightarrow \quad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) e^{-ik\omega_0 t} dt ,$$

where  $p = \frac{2\pi}{\omega_0}$  and  $\langle p \rangle$  denotes a suitable continuous interval of length  $p$ . For example,  $\int_{\langle p \rangle}$  can denote  $\int_0^p$ .

**Frequency Response of a DT LTI System** Consider a real, discrete-time LTI system having impulse response  $h : \mathbb{Z} \rightarrow \mathbb{R}$ . The frequency response  $H : \mathbb{R} \rightarrow \mathbb{C}$  of the system is given by:

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n} , \forall \omega .$$

**Q3.1 (20 Points)** (a) Consider a real, discrete-time system

$$F : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$$

having input signal  $x$  and corresponding output signal  $y$ , as described below:

$$\begin{cases} x(n) = \sum_{l=-\infty}^{\infty} \delta(n - 3l), & \forall n \\ y(n) = \sum_{l=-\infty}^{\infty} \delta(n - 2l), & \forall n. \end{cases}$$

(i) Could the system  $F$  be LTI? If your answer is "Yes," then determine as much as you can about the frequency response  $F$  of the system. Explain your reasoning succinctly, but clearly and convincingly.

(ii) Could the system  $F$  be memoryless? Explain your reasoning succinctly, but clearly and convincingly.

(b) Consider a real, discrete-time system

$$G : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$$

having input signal  $x$  and corresponding output signal  $y$ , as described below:

$$\begin{cases} x(n) = 1 + \cos\left(\frac{\pi}{3}n\right), \forall n \\ y(n) = 1 + \cos\left(\frac{\pi}{4}n\right), \forall n. \end{cases}$$

(i) Could the system be LTI? If your answer is "Yes," then determine as much as you can about the frequency response  $G$  of the system. Explain your reasoning succinctly, but clearly and convincingly.

(ii) Could the system be memoryless? Explain your reasoning succinctly, but clearly and convincingly.

(c) Consider a real, discrete-time system

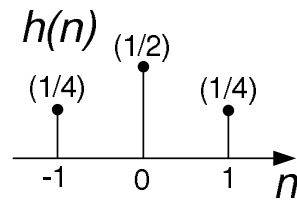
$$H : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$$

having input signal  $x$  and corresponding output signal  $y$ , as described below:

$$\begin{cases} x(n) = 1 + \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{2\pi}{3}n\right), \forall n \\ y(n) = \frac{1}{2} \cos\left(\frac{2\pi}{3}n\right), \forall n. \end{cases}$$

Could the system be LTI? If your answer is "Yes," then determine as much as you can about the frequency response  $H$  of the system. Explain your reasoning succinctly, but clearly and convincingly.

**Q3.2 (20 Points)** Consider a discrete-time LTI system whose impulse response  $h$  is shown below.



The impulse response sample values are zero for all  $|n| > 1$ . If the input  $x$  to the LTI system is

$$x(n) = \frac{1 + (-1)^n}{2} + \cos\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{\pi}{2}n + \frac{\pi}{5}\right), \quad \forall n,$$

determine an expression for  $y(n)$ , the sample values of the corresponding output signal  $y$ . Simplify your final expression. Explain your work succinctly, but clearly and convincingly.