EECS 20. Final Exam 15 May 1999

Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. **Write clearly and put a box around your answer.**

Print your name below

Last Name ______________ First ______________

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Problem 7

Total

1. **15 points** Answer these short questions and use the space below for your calculations.

   (a) The solutions of the equation $e^{j4\theta} = 1$ are $\theta =$

   (b) Express $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$:

   $$\cos 3\theta =$$

   $$\sin 3\theta =$$

   (c) For what *real-valued* numbers ω is the function x periodic:

   $$\forall n \in Ints,\ x(n) = \cos \omega n$$

   and what is the period?

   (d) The general form of the following matrix for $n \geq 0$ is:

   $$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n =$$

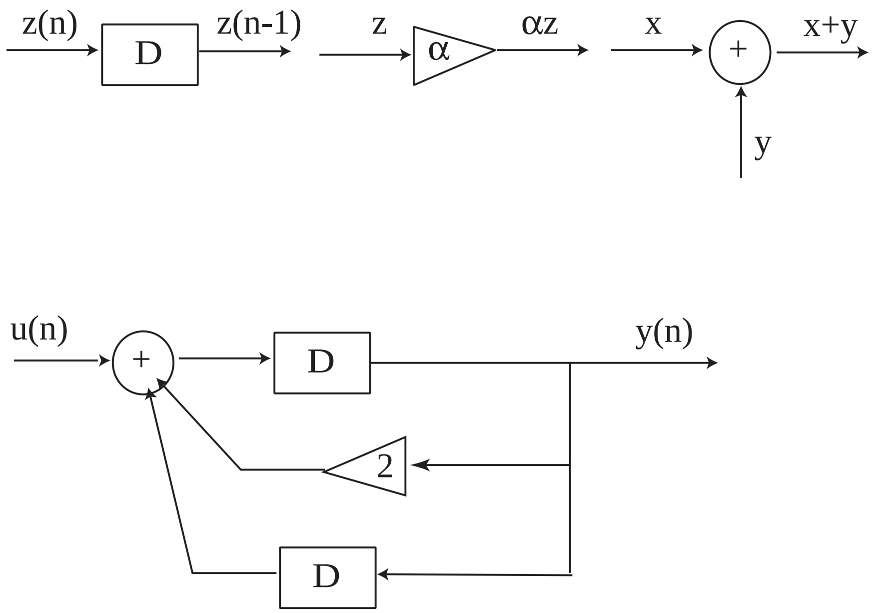

Figure 1: An LTI system can be built using unit delays, gains, and adders

2. **15 points** A LTI system can be built using unit delay elements D, gains α, and adders, shown on top of Figure 1.

   (a) Express the relation between the input and output of the system in the lower part of the figure in the form:

   $$y(n) = a_1 y(n-1) + \cdots a_k y(n-k) + b_1 u(n-1) + \cdots b_m u(n-m),$$

   i.e. determine k, m and the coefficients a_i, b_j for the system in the figure.

   (b) Determine the frequency response $H(\omega)$ of this system using the fact that $y = H(\omega)u$ when u is given by $\forall n,\ u(n) = e^{j\omega n}$.

3. **15 points** Consider the difference equation system:

$$\forall n,\ y(n) = 0.5y(n-1) + u(n-1).$$

(a) What is the zero-state impulse response of this system?

(b) Use this result to obtain the zero-state impulse response of the system:

$$\forall n,\ y(n) = 0.5y(n-1) + u(n-1) + u(n-2).$$

4. Consider the moving average system

$$\forall t \in \textit{Reals},\ y(t) = \int_{s=-0.5}^{0.5} x(t-s)ds.$$

(a) What is the impulse response h of this system?

(b) What is its frequency response?

(c) Use the previous result to determine the response y when the input is $\forall t,\ x(t) = \sin(\omega t)$.

5. **15 points** Let x be a continuous-time signal with Fourier Transform $X = FT(x)$, with

$$X(\omega) = \begin{cases} 1, & |\omega| < 2\pi \times 8,000 \text{ rads/sec} \\ 0, & \text{otherwise} \end{cases}$$

Let $y = \mathit{Sampler}_T(x)$, $Y = FT(y)$. Let $w = \mathit{IdealInterpolator}_T \circ \mathit{Sampler}_T(x)$, and $W = FT(w)$.

(a) Sketch X, Y, and W for $T = 1/20,000$ sec and $T = 1/12,000$ sec.

(b) For what values of T is $x = w$?

6. Construct a state machine with $U = Y = \{0, 1\}$ whose response function is: If $H(u) = y$, then

$$\forall n \geq 0,\ y(n) = \begin{cases} 0, & \text{if } u(n-3), u(n-2), u(n-1) = 000 \text{ or } 010 \\ 1, & \text{otherwise} \end{cases}$$

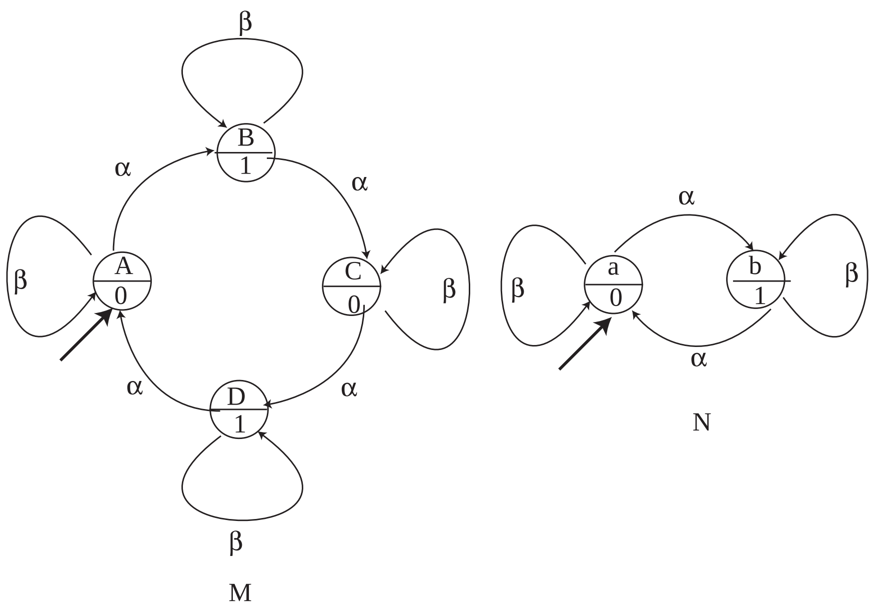

Figure 2: The machine N simulates machine M

7. Find a simulation relation S and show that N simulates M