Problem session, week 14

9.8 Suppose a discrete-time signal x has DTFT given by

$$X(\omega) = i \sin(K\omega)$$

for some positive integer K. Note that $X(\omega)$ is periodic with period 2π, as it must be to be a DTFT.

(a) Determine from the symmetry properties of X whether the time-domain signal x is real.

(b) Find x. **Hint**: Use Euler's relation and the linearity of the DTFT.

9.9 Consider a periodic continuous-time signal x with period p and Fourier series $X : \mathit{Ints} \to \mathit{Comps}$. Let y be another signal given by

$$y(t) = x(t - \tau)$$

for some real constant τ. Find the Fourier series coefficients of y in terms of those of X.

9.10 Consider the continuous-time signal given by

$$x(t) = \frac{\sin(\pi t/T)}{(\pi t/T)}.$$

Show that its CTFT is given by

$$X(\omega) = \begin{cases} T, & \text{if } |\omega| \leq \pi/T \\ 0, & \text{if } |\omega| > \pi/T \end{cases}$$

The following fact from calculus may be useful:

$$\int_a^b e^{c\omega} c d\omega = e^{cb} - e^{ca}$$

for real a and b and complex c.

9.11 If x is a continuous-time signal with CTFT X, then we can define a new time-domain function y such that

$$\forall\, t \in \mathit{Reals}, \quad y(t) = X(t).$$

That is, the new time domain function has the same shape as the frequency domain function X. Then the CTFT Y of y is given by

$$\forall\, \omega \in \textit{Reals}, \quad Y(\omega) = 2\pi x(-\omega).$$

That is, the frequency domain of the new function has the shape of the time domain of the old, but reversed and scaled by 2π. This property is called **duality** because it shows that time and frequency are interchangeable. Show that the property is true.

9.12 Use the results of exercises and to show that a continuous time signal x given by

$$x(t) = \begin{cases} T, & \text{if } |t| \leq \pi/T \\ 0, & \text{if } |t| > \pi/T \end{cases}$$

has CTFT X given by

$$X(\omega) = 2\pi \frac{\sin(\pi\omega/T)}{(\pi\omega/T)}.$$