Problem session, week 3

5 (Chapter 1, ex. 5 (revised)) **E** Use Matlab to plot the graph of the following continuous-time functions defined over $[-1, 1]$, and on the same plot display 11 uniformly spaced samples (0.2 seconds apart) of these functions.

(a) $f: [-1, 1] \to$ *Reals*, where for all $x \in [-1, 1]$, $f(x) = e^{-x} sin(10\pi x)$.

(b) *Chirp*: $[-1, 1] \to$ *Reals*, where for all $x \in [-1, 1]$, *Chirp*$(t) = cos(10\pi t^2)$.

2 (Chapter 2, ex. 2 (revised)) **T** This problem studies the relationship between the notion of **delay** and the graph of a function.

(a) Consider two functions f and g from *Reals* into *Reals* where $\forall\ t \in$ *Reals*, $f(t) = t$ and $g(t) = f(t - t_0)$, where t_0 is a fixed number. Sketch a plot of f and g for $t_0 = 1$ and $t_0 = -1$. Observe that if $t_0 > 0$ then *graph*(g) is obtained by moving *graph*(f) to the right, and if $t_0 < 0$ by moving it to the left.

(b) Show that if f: *Reals* $\to$ *Reals* is any function whatsoever, and $\forall\, t,\ g(t) = f(t - t_0)$, then if $(t, y) \in$ *graph*(f), then $(t + t_0, y) \in$ *graph*(g). This is another way of saying that if $t_0 > 0$ then the graph is moved to the right, and if $t_0 < 0$ then the graph is moved to the left.

(c) If t represents time, and if $t_0 > 0$, we say that g is obtained by *delaying* f. Why is it reasonable to say this?

6 (Chapter 2, ex. 6 (revised)) **C** A router in the Internet is a switch with several input ports and several output ports. A packet containing data arrives at an input port at an arbitrary time, and the switch forwards the packet to one of the outgoing ports. The ports of different routers are connected by transmission links. When a packet arrives at an input port, the switch examines the packet, extracting from it a destination address d. The switch then looks up the output port in its routing table, which contains entries of the form $(d,$ *outputPort*$)$. It then forwards the packet to the specified output port. Thus, the internet works by setting up the routing tables in the routers.

Consider a simplified router with one input port and and two output ports, named O_1, O_2. Let D be the set of destination addresses.

(a) Explain why the routing table can be described as a subset $T \subset D \times \{O_1, O_2\}$.

(b) Is it reasonable to constrain T to be the graph of a function from $D \to \{O_1, O_2\}$? Why?

(c) Assume the signal at the input port is a sequence of packets. How would you describe the space of input signals to the router and output signals from the router?

(d) How would you describe the switch as a function from the space of input signals to the space of output signals?

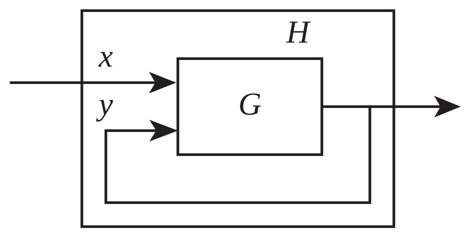
11 (Chapter 2 (new problem)) **T** Let $D = DiscSignals = [Ints \to Reals]$ and let

$$G\colon D \times D \to D$$

such that for all $x, y \in D$ and for all $n \in Ints$,

$$(G(x,y))(n) = x(n) - y(n-1).$$

Now suppose we construct a new system H as follows:



Define H (as much as you can).