Example: In section 2.3.2 we considered a simple **moving average** example where the output y is given in terms of the input x as

$$\forall\, n \in \textit{Ints}, \quad y(n) = (x(n) + x(n-1))/2.$$

The general form of this is the M-point moving average, where

$$\forall\, n \in \textit{Ints}, \quad y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k).$$

Comparing to (5.22), this is recognizable as a convolution with

$$h(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1/M & \text{if } 0 \le n < M \\ 0 & \text{if } n \ge M \end{cases}$$

This function, therefore, is the impulse response of an M-point moving average system. Notice that the impulse response is finite in extent (it starts at 0 and stops at $M-1$). For this reason, such a system is called a **finite impulse response system** or **FIR** system.

Example: Although moving-average systems are popular on Wall Street for smoothing out random price fluctuations, we will see in chapter 7 that they are not particularly good at this job. It is relatively easy to design much better smoothing systems.

The M-point moving average can be viewed as a special case of the more general FIR system given by

$$\forall\, n \in \textit{Ints}, \quad y(n) = \sum_{k=0}^{M-1} h(k)x(n-k).$$

Letting $h(k) = 1/M$ for $0 \le k < M$, we get the M-point moving average. Choosing other values for $h(k)$, however, we can get better smoothing (this will be explored in chapter 7).

We can write the general FIR system in the form of (5.31) and (5.32), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{bmatrix}, \; c = \begin{bmatrix} h(M-1) \\ h(M-2) \\ h(M-3) \\ \cdots \\ h(2) \\ h(1) \end{bmatrix}, \; d = h(0)$$

The $(M-1) \times (M-1)$ matrix A has coefficients $a_{i,i+1} = 1$, while all other coefficients are zero. The vector b has the last coefficient 1, while all others are zero. From this we note that the state vector is simply the $M-1$ past samples of the input,

$$s(n) = [x(n-M+1), x(n-M+2), \cdots, x(n-1)]^T,$$

a column vector. This is intuitive based on our notion of state. State, after all, is that which we must remember about the past.